

A Finite Time Analysis of an Ideal Applications to Absorption Refrigeration Systems.

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ABSTRACT

The characteristics of an irreversible steady-flow absorption refrigeration cycles are examined when approximated to a combination of ideal driving and cooling cycles. The optimized refrigeration cycle has then been combined with an optimized power cycle to produce a combined cycle. The general matching requirements of the two cycles are discussed. The cycles are then combined in a specific way such that they become a model for an ideal absorption refrigeration cycle. The influence of the design parameters on the performance of the combined cycle is analyzed. The performance of an experimental absorption unit is compared with the predictions made by the analysis.

KEYWORDS: Brayton cycle, Absorption refrigeration, Combined cooling and power

NOMENCLATURE

A	Heat Transfer Area
COP	Coefficient of Performance
h	Heat Transfer Coefficient
Q	Heat Transfer
S	Entropy
t	Time
T	Temperature
W	Work
y	Internal Temperature Ratio of the Cooling Cycle T_e/T_a
h	Efficiency
q	Heat Transfer Factor = hA

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SUBSCRIPTS

- a Cooling Cycle High Temperature Isotherm
- c Driving Cycle Low Temperature Isotherm
- C Driving Cycle Low Temperature Reservoir
- e Cooling Cycle Low Temperature Isotherm
- h Driving Cycle High Temperature Isotherm
- H Driving Cycle High Temperature Reservoir
- Max Maximum value
- o Cooling Cycle High Temperature Reservoir
- r Cooling Cycle Low Temperature Reservoir
- s system of combined cycles
- 1 Driving Cycle
- 2 Cooling Cycle

INTRODUCTION

Since the introduction of Finite Time Thermodynamics a great deal of work has been expended in examining the operational bounds of thermodynamic processes [1-8]. The interaction of energy transfer and degradation mechanisms with the configuration of power cycles is now well established so that it is possible to explain why practical cycles operate in a region between the two ideal limits of reversible maximum efficiency and irreversible maximum power.

The majority of the published work in this field has largely been related to power cycles but attention has also been given to cooling or refrigeration cycles notable the work of Tsirlin and his co-workers [8-12] who have analyzed the optimal performance of standalone refrigeration cycles. As the power supply to these cycles is not considered the analysis is perhaps more appropriate to process engineering rather than refrigeration. The present authors [13,14] produced numerical optimizations of a combined power and refrigeration cycle that were based in the first instance on maximizing the refrigeration effect and in the second on minimizing irreversibilities. It is the purpose of this work to produce an analytical solution of a combined power and refrigeration cycle and produce some design guidelines that can be applied to a specific combination of cycles that may be considered to be an ideal

_____ .A Finite Time Analysis of an Ideal Applications to Absorption Refrigeration Systems representation of an absorption refrigeration cycle. To this end, a refrigeration cycle is considered to be a combination of a Carnot power cycle and a Carnot cooling cycle with temperature differences imposed between the cycle isotherms and the reservoirs as illustrated in Figure 1. It is not intended to extend the science of Finite Time Thermodynamics in this work but rather, it is intended to illustrate the engineering utility of this type of analysis by comparing the results of the analysis with data taken from an experimental absorption refrigeration unit.

OPTIMIZATION OF THE DRIVING CYCLE

The analysis of a single Carnot power cycle is now well established for the non-flow cycle [1] and the steady flow cycle [15]. The rate of energy exchange between the cycle and the external thermal reservoirs is expressed in terms of a respective temperature difference and the first and second laws impose internal constraints on the Carnot cycle must also be satisfied. It can be shown that the temperature differences between the cycle and the hot and cold thermal reservoirs are respectively:

$$T_H - T_h = \frac{\sqrt{T_H}(\sqrt{T_H} - \sqrt{T_C})}{1 + \frac{\theta_h}{\theta_c}} \quad 1$$

$$T_C - T_c = \frac{\sqrt{T_C}(\sqrt{T_H} - \sqrt{T_C})}{1 + \frac{\theta_h}{\theta_c}} \quad 2$$

Further optimization by Bejan [16] using arguments drawn from the optimization of heat exchangers has shown that for a given heat exchange area, optimum conditions are met when the overall

q (q = $\theta_h + \theta_c$) is shared equally between the hot and the cold ends of the cycle, i.e. $\theta_h = \theta_c$.

OPTIMIZATION OF THE COOLING CYCLE

The analysis of the cooling cycle draws upon the previous analysis of the driving cycle. The processes within the internal Carnot cycle can be characterized by the following conditions:

1. The only irreversibilities in the cycle occur due to the finite rate of heat transfer between the cycle and the external reservoirs.

2. The isentropic processes occur in a time that is small compared to the isothermal processes. The time for a unit mass of working fluid to travel

around the cycle is then the sum of the residence times associated with the heat addition and heat rejection processes.

$$\tau = t_{h2} + t_{c2} \tag{3}$$

3. The heat transfer processes can be characterized by Newton’s law of cooling such that the energy transfers from and to the working fluid can respectively be represented by the following equations.

$$Q_{c2} = \theta_{c2}(T_r - T_e)t_{c2} \tag{4}$$

$$Q_{h2} = \theta_{h2}(T_a - T_o)t_{h2} \tag{5}$$

Considerations of the second law also require that the following equation be satisfied.

$$\frac{Q_{c2}}{Q_{h2}} = \frac{T_e}{T_a} = y \tag{6}$$

The average refrigeration rate in the cycle is then

$$\dot{Q} = \frac{Q_{c2}}{\tau} = \frac{\theta_{c2}(T_r - T_e)}{1 + \frac{t_{h2}}{t_{c2}}} \tag{7}$$

Substituting for t_{h2} and t_{c2} from Eqs. (4), (5) and (6) and rearranging produces the following relationship:

$$\dot{Q} = \frac{\theta_{c2} \left[1 + \frac{1}{y} \right]}{\frac{1}{\theta_{c2}(T_r - T_e)} + \left[\frac{\theta_{c2}}{\theta_{h2}} \right] \frac{1}{(T_e - \gamma T_o)}} \tag{8}$$

If this equation is differentiated to determine a maximum refrigeration rate as with the power cycle an unbounded solution will result due to the relative position of the cycle isotherms and the reservoirs. A satisfactory result can be obtained however if this equation is differentiated with respect to T; with y as a constant. This is in effect maintaining the COP of the refrigeration cycle constant and will produce the following expression for $T_1 - T_2$ as shown below in Eq. (9). This then leads to an expression for the optimum refrigeration

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rate as shown in Eq. (10).

$$T_r - T_e = \frac{T_r - yT_o}{1 + \left(\frac{\theta_{c2}}{\theta_{h2}}\right)^{1/2}} \quad 9$$

$$(\dot{Q}_{c2})_{opt} = \frac{\theta_{c2}(T_r - yT_o)}{1 + \left(\frac{\theta_{c2}}{\theta_{h2}}\right)^{1/2}} \quad 10$$

The temperatures of the isotherms of the cooling cycle can also be evaluated as shown in Eqs. (11) and (12).

$$T_e = \frac{yT_o - T_r \left(\frac{\theta_{c2}}{\theta_{h2}}\right)^{1/2}}{1 + \left(\frac{\theta_{c2}}{\theta_{h2}}\right)^{1/2}} \quad 11$$

$$T_a = yT_e \quad 12$$

These relationships are examined further in a later section to establish the influence of y on the maximum refrigeration rate. The above expression for the refrigeration rate is very similar in form to that of the optimum power output of a driving cycle. Following Bejan [16], further optimization can be performed by proper distribution of the heat transfer factors between the cycle and the reservoirs. If the refrigeration effect is non-dimensionalised by dividing it by the product of T_r and the sum of the heat transfer factors the resulting equation can be optimized with respect to the ratio of the heat transfer factors. Such an analysis will lead to the result shown in Eq. (13) which when introduced into Eq. (10) will lead. to the ultimate maximum refrigeration effect as shown in Eq. (14).

$$\theta_{h2} = \theta_{c2} \quad 13$$

$$(\dot{Q}_{c2})_{opt} = \frac{\theta_{c2}(T_r - yT_o)}{2} \quad 14$$

MATCHING CONDITIONS

The following section examines the implications of combining a power and. a cooling cycle whenthey are both optimized for maximum energy trans-

fer rate. The following draws on the analysis of the single power cycle reported in Ref. [15]. The energy transfer rate to the power cycle is as follows:

$$Q_{h1} = \frac{\theta_1 T_H \left\{ 1 - \sqrt{\frac{T_C}{T_H}} \right\}}{2} = \frac{\theta_1 T_H \eta_1}{2} \quad 15$$

And the rate at which this is delivered to the cooling cycle is shown in Eq. (16).

$$\dot{Q}_{h1} \eta_1 = Q_{h2} - Q_{c2} = \frac{\theta_1 T_H \eta_1^2}{2} \quad 16$$

Since from the second law

$$\frac{Q_{h2}}{T_a} = \frac{Q_{c2}}{T_c} \quad 17$$

Then:

$$\eta_1 = \left[\frac{\theta_2}{\theta_1} \right]^{1/2} \left[\frac{T_r \left(\frac{1}{y} - 1 \right) + T_o (y-1)}{T_H} \right]^{1/2} \quad 18$$

The energy input rate to the driving cycle is then:

$$Q_{h1} = (\theta_1 \theta_2)^{1/2} \frac{[T_H T_r \left[\frac{1}{y} - 1 \right] + T_o T_H (y-1)]^{1/2}}{2} \quad 19$$

The expression for the coefficient of performance of the combined cycles can then be represented as shown in Eq. (20).

$$COP_S = \frac{Q_{c2}}{Q_{h1}} = \left[\frac{\theta_2}{\theta_1} \right]^{1/2} \left\{ \frac{T_r - y T_o}{[T_H T_r \left(\frac{1}{y} - 1 \right) + T_H T_o (y-1)]^{1/2}} \right\} \quad 20$$

This can be simplified to produce the following expression:

$$COP_S = \left(\frac{\theta_2}{\theta_1} \right)^{1/2} \frac{y^{1/2} (T_r - y T_o)^{1/2}}{T_H^{1/2} (1-y)^{1/2}} \quad 21$$

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If the analysis performed on Eq.(12) is applied to Eq.(21) to determine the optimum arrangement of heat transfer factors between the two cycles it can be shown that this condition is also reached when that is the heat transfer factors are equal for all the heat transfer processes in the combined cycle and the cycle COP becomes as shown below.

$$COP_S = \left[\frac{y(T_r - yT_o)}{T_H(1-y)} \right]^{1/2} \quad 22$$

At this condition the optimum refrigeration effect then becomes as shown in Eq.(23).

$$(\dot{Q}_{C2})_{opt} = \frac{\theta(T_r - yT_o)}{2} \quad 23$$

Rearranging of Eq.(10) leads to the following constraint on the temperatures of the cooling cycle:

$$\frac{T_r}{T_e} + \frac{T_o}{T_a} = 2 \quad 24$$

The above analysis has effectively decoupled the system COP from the heat transfer process as the COP can be determined from consideration of the cycle temperatures alone with the heat transfer factors influencing only the energy exchange rates.

DESIGN CONSIDERATIONS

An ideal absorption refrigeration cycle can be considered to be a combination of a Carnot driving and cooling cycles, Ref [17]. The previous analysis has illustrated the relationships between the temperature ratios of the driving and cooling cycles for the condition of maximum refrigeration effect but show very little information about the overall COP of the cycles. It is the objective of the following section to explore the conditions that the optimization process places on the overall COP and thus produce some guidelines that may be of interest to designers of absorption refrigeration machines. The machine to be examined can be considered to be air cooled having a common heat rejection reservoir between the driving and cooling cycles with T_c equal to T_o . Combining Eqs.(15), (22) and (23) and applying the above condition leads to:

$$\frac{(T_r - yT_c)}{T_H \eta_1} = \left[\frac{y(T_r - yT_c)}{T_H(1-y)} \right]^{1/2} \quad 25$$

Which can be rearranged to produce the following:

$$\left[\frac{T_H}{T_C}\right]^{1/2} = \left\{\left(\frac{T_r}{T_C} - y\right)\left(\frac{1-y}{y}\right)\right\}^{1/2} + 1 = \frac{T_h}{T_c} \quad 26$$

Equation (10) can now be utilized to produce a similar relationship for T_e/T_c :

$$\frac{T_e}{T_c} = \frac{\frac{T_r}{T_C} + y}{2} \quad 27$$

The COP of the combined cycle is the product of the efficiency of the driving cycle and the cop of the cooling cycle, i.e.

$$COP_S = \eta_1 COP_2 = \left[1 - \sqrt{\frac{T_C}{T_H}}\right] \frac{y}{1-y} \quad 28$$

The matching considerations of the combined cycles results in the efficiency of the driving cycle decreasing as the COP of the cooling cycle increases. Combining Eqs. (22) and (26) produces the following expression for the overall COP.

$$COP_S = \left(\frac{y}{1-y}\right)^{1/2} \frac{\left(\frac{T_r}{T_C} - y\right)^{1/2}}{\left\{\left(\frac{T_r}{T_C} - y\right)\left(\frac{1-y}{y}\right)\right\}^{1/2} + 1} \quad 29$$

The behavior of the above expression is shown in figure 2 for constant values of T_r/T_o . It can be seen that a maximum value of the COP is exhibited. The loci of these maximum values are shown plotted in figure 3 together with the corresponding values of T_c/T_H , T_e/T_c and T_r/T_c . Also shown plotted in figure 3 are some data obtained from an experimental air cooled Lithium—Bromide absorption refrigeration unit. Full details of this experimental facility can be obtained from [18]. The rig has been designed to enable the component temperatures and reservoir temperatures (to use the current terminology) and the various flow rates of the absorbent and refrigerant to be varied over a wide range of operating conditions. The results shown in figure 3 – represent the COPs obtained using the component and reservoir temperatures indicated from the theoretical analysis.

DISCUSSION OF THE RESULTS

The above analysis clearly illustrates the two extreme operating conditions of a refrigeration cycle, zero refrigeration rate with zero COP and zero refrigeration rate with a maximum COP equal to the Carnot COP for the given

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reservoir temperatures. It has been shown that the refrigeration effect of an endo-irreversible Carnot cycle is a function of the temperatures of the reservoirs but it is also dependent upon the internal temperature ratio T_e/T_a . The COP of the combined cycle can be described by the reservoir temperatures of the driving cycle T_H/T_c and γ the internal temperature ratio of the cooling cycle. If the internal temperature ratio is equal to the reservoir temperature ratio, producing a fully reversible cooling cycle, the refrigeration rate will be zero but the coefficient of performance of the cycle will be equal to the maximum possible value for the reservoir temperature ratio. Conversely reducing the temperature ratio T_e/T_c reduces the refrigeration rate until the cycle will cease to operate producing the point of zero COP and zero refrigeration rate. The characteristics of the driving cycle have a significant effect on the characteristics of the cooling cycle. A high temperature heat source of the driving cycle gives the possibility of obtaining low refrigeration temperatures but at the expense of low COP. High COPs can be obtained when low temperature heat sources are utilized in the power cycle. The actual operating characteristics of the driving and cooling cycles must therefore take into account the performance requirements of the combined cycle. These findings mirror the operation of a single driving cycle for which fully reversible operation will produce zero power output. Unlike the driving cycle however in which the power output and the efficiency are both determined by the reservoir temperatures the cooling cycle requires an additional constraint to define its operation. This can be in the form of a coefficient of performance if the refrigeration effect is to be determined or a refrigeration effect if the coefficient of performance is not specified.

The difficulty of producing very low temperatures and the unattainability of absolute zero with this type of refrigeration cycle are illustrated. The design analysis clearly shows that a single effect absorption refrigeration machine is best suited to working with a low-grade heat source such as a waste heat recovery mode rather than being directly fired. The corollary to this is that performance penalties are met when direct fired absorption units are used in a waste heat recovery mode thus a CHP/absorption refrigeration combined cycle must be equipped with an absorption unit that is optimized for the specific thermal environment to which it will be exposed. The experimental values of COP of the experimental single effect absorption unit are seen to be smaller

than the theoretical values indicated in figures 2 and 3. This is due to internal entropy generation processes that are not considered in the ideal model considered here. Towards high values of y the COP deviated significantly from the theoretical values due it is thought to internal heat transfer effects as it proved very difficult to stabilize the unit with values of y greater than 0.9

CONCLUSIONS

The operating characteristics of a combination of a driving and cooling cycle, optimized for maximum output has been examined.

The influence of the power cycle on the performance of the combined cycle has been demonstrated. The optimal system COP ((COPs)Max) is bounded by two extreme cases the first being zero cooling effect zero COP and the second zero cooling effect with a maximum COP when both the driving and cooling cycles are operating reversibly. The requirement for careful matching of the driving and cooling cycles to obtain a specified combined cycle performance and optimum-operating performance for a given environmental situation is clearly demonstrated in this analysis.

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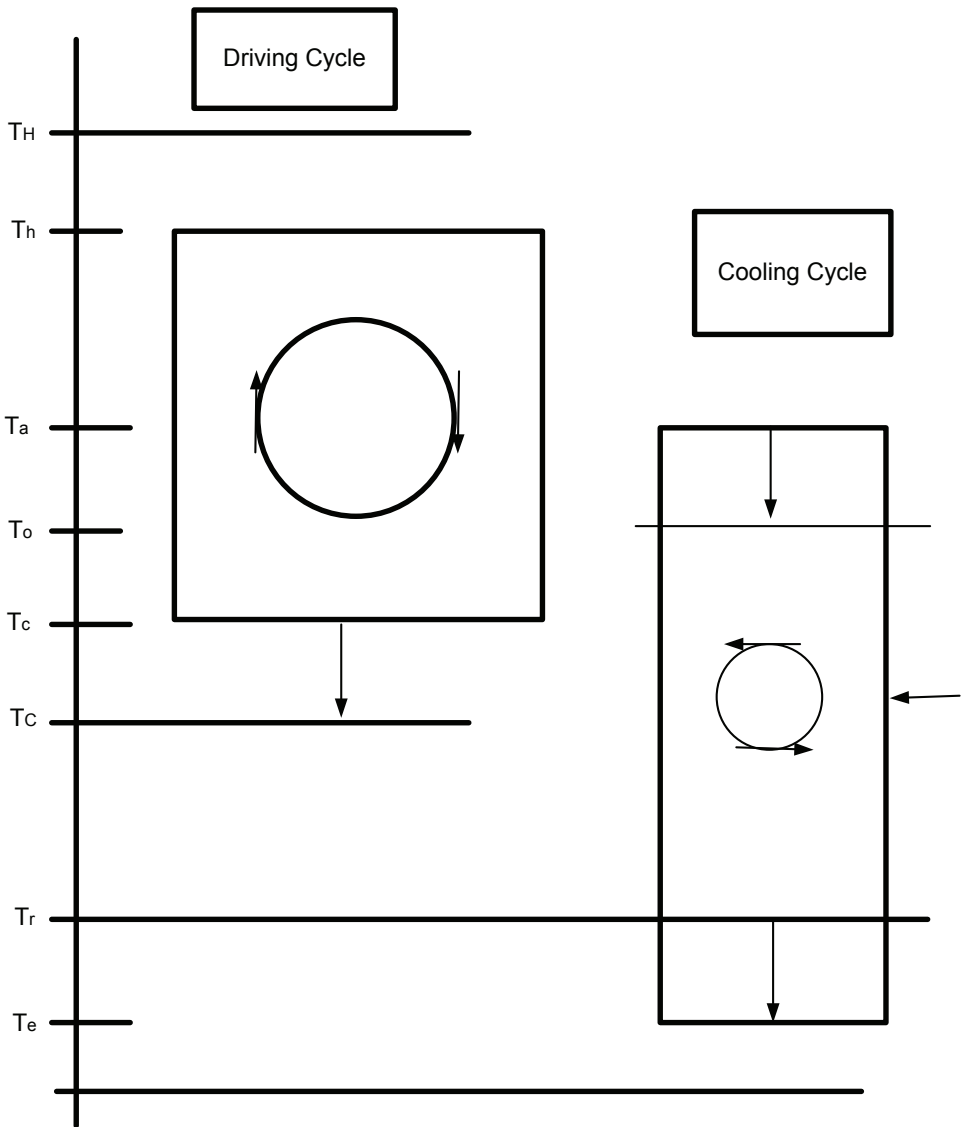


Figure.1 T-S diagram of the Combined Cycle.

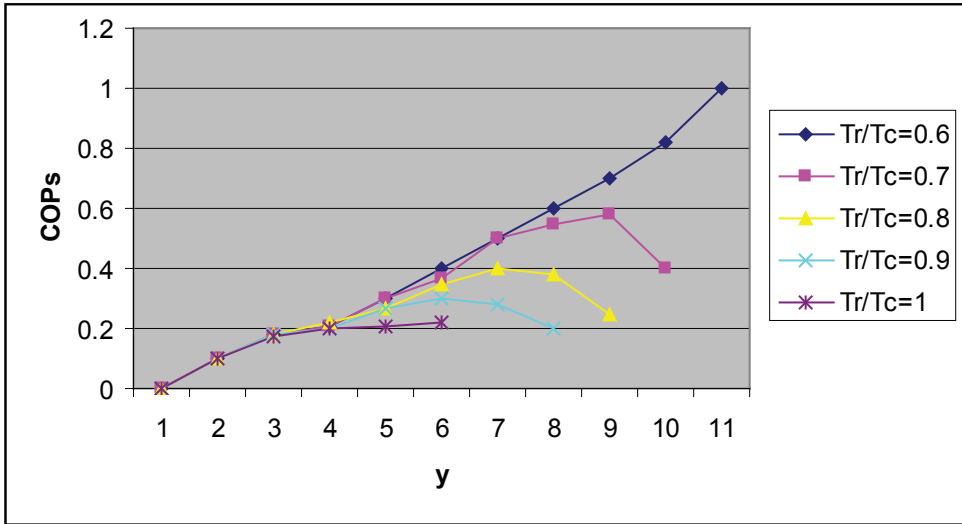


Figure.2 The Relationship between the Overall System COP and Y for specified Tr/Tc at Maximum Cooling Rate. $T_o = T_c$.

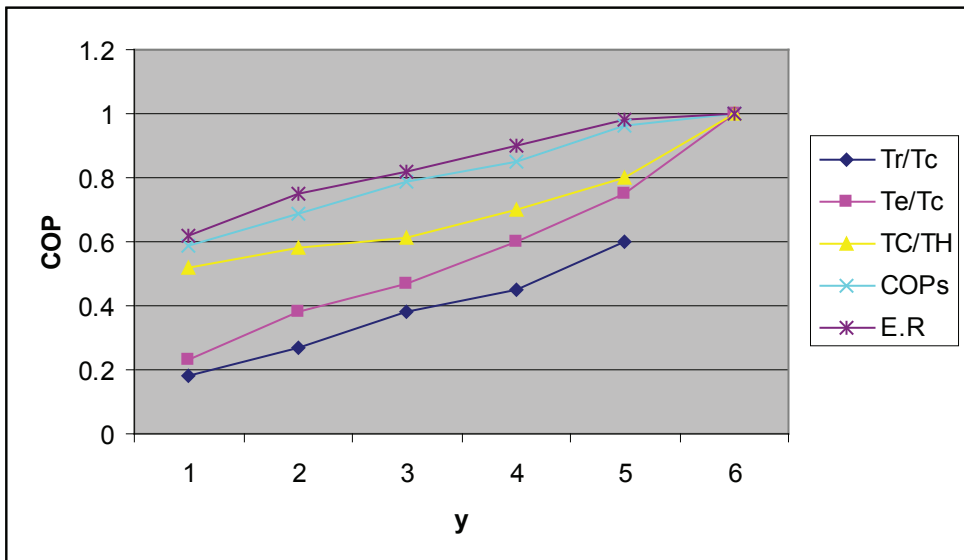


Figure.3 The Relationship between the Maximum System COP, the cycle temperature ratios and Y.

تحليل زمني محدود للمثل الأعلى لتطبيقات أنظمة امتصاص التبريد

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المقدمة:-

يتم اختبار الخصائص التدفق ثابت الانعكاسي لدورات التبريد الامتصاصية عند تقارب مجموعة توافقية مكونة من الدورة المثالية ودورات التبريد. دورة التبريد، اما القصوى ستشترك مع دورة القدرة القصوى لإنتاج دورة مشتركة. وفي هذا التطبيق قد تمت مناقشة المتطلبات العامة للدورتين المتوائمتين، وستشترك الدورتين بطريقة محددة بحيث تصبح نموذجا لدورة التبريد الامتصاصية المثالية. تم تحليل تأثير البارامترات التصميمية على أداء الدورة المشتركة، وأن أداء وحدة الامتصاص الاختبارية قد تمت مقارنته مع التنبؤات الناتجة من التحليل في هذا الدراسة.

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