Nouvel Approach for Better System Performance with Reduced Order .

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Abstract:

The paper considers a problem of model reduction of high order sampled systems. The computation algorithm to obtain a reduced order model from a given high order pulse transfer function is presented. The method is based on matching the weighting sequence of the original system with that of the reduced model. The generalized least squares method is then used for determination the model parameters.

1. Introduction:

Model reduction receives growing attention, in the mathematics community and in various application areas. For example, in electronics reduced order models are used to capture the behavior of complicated interconnect systems in the form of a small electronic circuits. These small circuits can be coupled to an already existing circuit and co-simulated with it, so as to obtain a better description of the system consisting of the circuit and their interconnects. Similar approaches are used in other application areas, the common denominator being the mathematical techniques for model reduction . Also desirable to use models of low order for the purpose of the computer simulation and/or design of control system .

A number of investigations have been carried out in approximating high-order linear systems to low order ones [1], [2]. The pade approximation method [3], [4] is very popular among the proposed methods of model reduction. It is conceptually attractive and has computational advantages over many other methods. However, this method has the drawback that it may produce an unstable reduced model although the original system is stable [5]. The squared magnitude pade approximation has been proposed by many authors [6], [7], [8]. In [9], model reduction is performed using continued fraction.

The authors in [10] have used the tangent phase pade approximation for order reduction of pulse transfer function through the bilinear transformation $z = \frac{1+s}{1-s}$. However, this technique is very tedious.

In this paper, the model of a high order pulse transfer function is performed through two stages. First, the weighting sequence of the original system is computed and matched with that of the reduced model. Second, a set of linear algebraic equations have been solved using the generalized least squares method.

2. Model Reduction of Pulse Transfer Functions :

Consider a high order sampled data system given by the following pulse transfer function of n^{th} order

$$G_n(z^{-1}) = \frac{a_0 + a_1 z^{-1} + \dots + a_n z^{-n}}{1 + b_1 z^{-1} + \dots + b_n z^{-n}} \qquad (1)$$

Assuming that the poles of the system are within the unit circle of the z-plane. The problem is to obtain a model $\hat{G}_m(z^{-1})$

of order $m < n$

$$G_m(z^{-1}) = \frac{\alpha_0 + \alpha_1 z^{-1} + \dots + \alpha_m z^{-m}}{1 + \beta_1 z^{-1} + \dots + \beta_m z^{-m}}$$

$$(m < n) \qquad (2)$$

Which approximate the original system (1).

The system in (1) can be expanded as a weighting sequence $\{h_i, i = 0, 1, 2, ..., p\}$ as follows

$$G_n(z^{-1}) = h_0 + h_1 z^{-1} + h_2 z^{-2} + \dots + h_p z^{-p} \quad (p > 2n) \qquad (3)$$

To match the weighting sequence of the original system (1) with that of the model (2), it is necessary that

$$\frac{\alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \dots + \alpha_m z^{-m}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2} + \dots + \beta_m z^{-m}} = h_0 + h_1 z^{-1} + \dots + h_p z^{-p}$$

Then the following linear algebraic equations can be obtained

$$\alpha_i = \sum_{j=0}^{i} \beta_j h_{i-j} \qquad (i = 0, 1, 2, ..., m) \qquad (4)$$

or in vector-matrix form

$$\begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix} = \begin{bmatrix} h_0 & & & \\ h_1 & h_0 & & \\ h_2 & h_1 & h_0 & \\ h_m & h_{m-1} & \dots & h_0 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix} \qquad (5)$$

and

$$
\begin{aligned}
h_m\beta_1 + h_{m-1}\beta_2 + \ldots + h_1\beta_m &= -h_{m+1} \\
h_{m+1}\beta_1 + h_m\beta_2 + \ldots + h_2\beta_m &= -h_{m+2} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
h_{p-1}\beta_1 + h_{p-2}\beta_2 + \ldots + h_{p-m}\beta_m &= -h_p
\end{aligned} \qquad (6)
$$

or in the vector – matrix form

$$
\underset{H}{\begin{bmatrix} h_m & h_{m-1} & h_{m-2} \ldots h_1 \\ h_{m+1} & h_m & h_{m-1} \ldots h_2 \\ h_{m+2} & h_{m+1} & h_m \ldots h_3 \\ \vdots & \vdots & \vdots \ \ldots \vdots \\ h_{p-1} & h_{p-2} & h_{p-3} \ldots h_{pm} \end{bmatrix}_{(p-m)\times m}}
\underset{\beta}{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_m \end{bmatrix}_{m\times 1}}
= -
\underset{h}{\begin{bmatrix} h_{m+1} \\ h_{m+2} \\ h_{m+3} \\ \vdots \\ h_p \end{bmatrix}_{(p-m)\times 1}} \qquad (7)
$$

Equation (7) can be solved for the β - parameters using the generalized least squares method. The least square solution of (7) is

$$\beta = \left[H^T\ H\right]^{-1} H^T.h \qquad (8)$$

where "T" denotes for a matrix transpose.

Having the denominator of the reduced model determined, the parameters of the numerator of, the α parameters can be computed using the formula (4) or (5).

3. The Proposed Model Reduction Algorithm :

The reduction procedure can be summarized by the following steps:

Step 1: compute the weight sequence of the original system (1)

$$\{h_i,\ i = 0,\ 1,\ 2, \ldots, p\}, \quad p > 2n$$

Step 2: construct the matrices H and h using (7)

Step 3: compute β -parameters of the reduced model using (8)

Step 4: compute the α -parameters of the reduced model using (4,5)

Example

To illustrate the applicability of the proposed reduction procedures, let us consider the following pulse transfer function of 4^{th} order system given in [8].

$$G_4(z) = \frac{0.07844z^3 - 0.1556z^2 + 0.1042z - 0.02388}{z^4 - 2.698z^3 + 2.643z^2 - 1.106z + 0.1653} \qquad (9)$$

Fig. 1 shows the unit step response of the original 4^{th} order system. A 2^{rd} order model is obtained using the proposed method given by

$$\hat{G}_2(z) = \frac{0.0448z - 0.03733}{z^2 - 1.822z + 0.8327} \qquad (10)$$

The unit step response of the 2^{nd} order model is shown in Fig. 2.

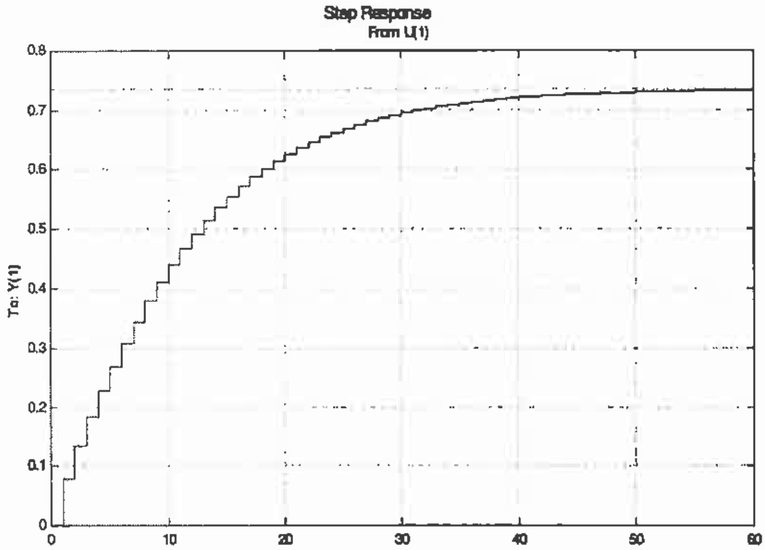

Fig.1 unit step response of the original 4^{th} order system

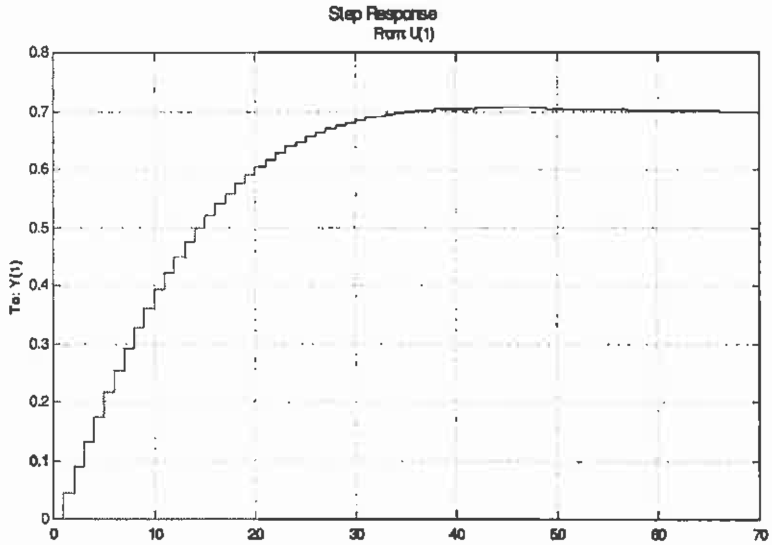

Fig.2 Unit step response of the 2^{nd} order model

4. Conclusions :

A reduction of a reduced order models for sampled data systems is proposed. By computation, it can be reported in a digital computation and possible extension to multivariable system .

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ملخص البحث :

هذه الورقة تدرس مشكلة تقليل رتبة الانظمة , استخدمت الحسابـــات اللوغارتمية للحصـول على نموذج نظام بالرتبة القليلة من النظام بالرتبـــة العالية باســتخدام طريقة المربعات الصغيرة تم العمل والتحليل لايجاد بارامتريات النموذج.