DUALITY THEORY IN PRODUCTION
THEORETICAL APPROACH

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DUALITY THEORY IN PRODUCTION

I- INTRODUCTION

In agriculture farmers are always confronted with Making decisions about what and how to produce. In most cases, decisions turn out to be that farmers would be engaged in more than one agricultural enterprise.

Past research has been devoted to estimating supply functions for many agricultural commodities by Simply estimating a single supply function by applying an "OLS" estimate. Using this method, ignore the fact that most farmers, in reality, produce multiple outputs by using multiple inputs.

In general, supply and factor demand functions can be derived by two approaches; (1) primal approach and (2) dual approach. Both approaches will be discussed in section IV of this paper.

Dual approach will be used in this paper to estimate two supply functions (livestock and livestock product, and all crops) and four factor demand functions (feed, seed, fertilizer, and hired farm labor) for 15 years time period.

II- OBJECTIVE OF THE PAPER

The main objective of this paper is to estimate profit share equations by using an econometric model based on duality theory. Other objectives to be derived from the estimated models are:

1- Own - Price Elasticities.
2- Cross-Price Elasticities.
3- Measure of Return To Size.
4- Measure of Technological Change.
5- Input /output Economic Relationships.

III- REVIEW OF THE THEORY

Let $F(Y, X / Z) = 0$ be the implicit production function which is also called a transformation function. Y and X are defined as vectors of outputs and inputs, respectively.

$Y = (y_1, y_2, \dots y_n)$, $X = (x_1, x_2, \dots x_m)$ where y_i is the quantity of output i produced and x_j is the quantity of input j used in the production process, Where Z is defined as fixed inputs.

The following assumptions are made in this study:

1- Perfect competition in output and input markets.
2- F is continuous and twice differentiable.
3- F is strictly convex set.
4- F is closed and bounded in Y and X of the positive orthant.
5- F is monotonic which means that if X is in F (Y, X) and $X' \geq X$, then x' is also in F (Y, X).
6- F is a regular set.
7- Farmers are profit maximizers.

Profit can be defined as

$$\pi = TR - TVC - TFC \text{ -----------------(1)}$$

$$\pi = PY' - RX' - TFC \text{ -------------------(2)}$$

Where TR = Total Revenue, TVC = Total Variable Cost, and TFC = Total Fixed Cost.

P is defined as a vector of output prices, where $P = (P_1, P_2, \dots P_n)$ and R is defined as an input prices vector where $R = (r_1, r_2, \dots r_n)$.

Primal Approach

Supply and factor demand functions are obtained in the following way:

$$\text{Maximise } \pi = PY' - RX' - TFC + \lambda F(Y, X / Z) \text{ .....(3)}$$

The first order condition is:

$$\frac{d\pi}{dY} = P + \lambda \frac{dF}{dX} = 0 \text{------------- (4)}$$

$$\frac{d\pi}{dX} = -R + \lambda \frac{dF}{dX} = 0 \text{ ------------(5)}$$

$$\frac{d\pi}{d\lambda} = F(Y, X/Z) = 0 \text{ ------------ (6)}$$

where

$$\frac{d\pi}{dY} = \left(P_1 + \lambda \frac{dF}{dY_1}, P_2 + \lambda \frac{dF}{dY_2} \dots, P_n + \lambda \frac{dF}{dY_n}\right) = (0, 0 \dots .0)_{1xn}$$

$$\frac{d\pi}{dX} = \left(-r_1 + \lambda \frac{dF}{dX_1}, -r_2 + \lambda \frac{dF}{dX_2} \dots, -r_m + \lambda \frac{dF}{dX_m}\right) = (0, 0 \dots .0)_{1xm}$$

From (4) and (5) we get the following relations:

$$\frac{Pi}{Pj} = \frac{dF/dyi}{dF/dYj} = \frac{-dyj}{dyi} = RPT_{yi\,yj} \dots \dots \dots (7)$$

$$\frac{ri}{rj} = \frac{dF/dxi}{dF/dxj} = \frac{-dxj}{dxi} = RTS_{xi\,xj} \dots \dots \dots (8)$$

$$\frac{ri}{Pk} = \frac{-dF/dxi}{dF/dyk} = \frac{dyk}{dyi} = \dots \dots \dots \dots \dots (9)$$

Were Relation (7) states that the rate of product transformation for every pair of outputs (RPTyiyj), holding the levels of all other inputs and outputs constant, must equal the ratio of their prices.

Relation (8) states that the rate of technical substitution for every pair of inputs ($RTS_{xi,\,xj}$) holding the level of all other inputs and outputs constant, must equal their price ratio.

Relation (9) can be rewritten in another form as

$$r_i = P_k \frac{dy_k}{dx_i} \dots \dots \dots \dots (10)$$

which says that the value of the marginal productivity of each input, with respect to each output. must be equal to the input price.

All three relations (7, 8, 9) are necessary conditions for profit maximization. The second order condition for profit maximization requires that the determinant of the border Hessian matrix (H^B) must be negative semi-definite. H^B can be written in the Following form:

$$H^B = \begin{bmatrix} \frac{d^2\pi}{d\lambda^2} & \frac{d^2\pi}{d\lambda dx} & \frac{d^2\pi}{d\lambda dy} \\ \frac{d^2\pi}{dx\, d\lambda} & \frac{d^2\pi}{dx^2} & \frac{d^2\pi}{dx\, dy} \\ \frac{d^2\pi}{dy\, d\lambda} & \frac{d^2\pi}{dy\, dx} & \frac{d^2\pi}{dy^2} \end{bmatrix}_{(n+m+1)\times(n+m+1)} \quad ....(11)$$

From (11), negative semi-difiniteness of H^B requires that the relevant border Hessian determinants alternate in signs.

$$|H_2{}^B| > 0 \quad |H_3{}^B| < 0 \quad |H_4{}^B| > 0 ....$$

Therefore Supply and factor demand functions are obtained by Solving the behavioral equations (4, 5, 6).

$$Y^* (P, R) = [Y^*_1 (P, R), Y^*_2 (P, R), ..., Y^*_n (P, R)] \quad .....(12)$$

$$X^* (P, R) = [X^*_1 (P, R), X^*_2 (P, R), ..., X^*_m (P, R)] \quad .....(13)$$

The indirect profit function (IPF) is obtained by substituting the profit maximizing supply and factor demand functions (12,13) into inflowing equation. The (IPF) can be written as follow

$$\pi = PY' - RX' - TFC \text{ -----------------} (2)$$

$$\pi^* (P, R,F) = PY^* (P, R) - RX^* (P, R) - TFC .....(14)$$

The indirect profit function is a function of the output and input prices that give the maximum level of profit at different alternative price levels.

Dual Approach:

Duality theory offers a convenient way of deriving and estimating supply function and factor demand function that are consistent with the theoretical properties of production function. Duality concept was first introduced into the economic analysis by Hot telling (in 1932) in the consumer theory. Shepherd (in 1953) extended the use of duality theory into the cost theory. McFadden generalized the use of duality in the production theory to include profit and revenue functions.

Supply and factor demand functions are obtained by applying Hottelling Lemma to the indirect profit function (14).

$$\frac{d\pi^*(P,R)}{dp} = Y^*(P,R) \quad ....(15)$$

$$\frac{d\pi^*(P,R)}{dR} = -X^*(P,R) \qquad \ldots\ldots(16)$$

The indirect profit function is characterized by the following properties.

(1) $\Pi^*(P, R)$ is continuous and twice differentiable.

(2) $\Pi^*(P, R)$ is non decreasing in the output prices.

If $p^0 \geq p$, then $\Pi^*(P^0, R) \geq \Pi^*(P,R)$.

(3) $\Pi^*(P,R)$ is non increasing in the input prices.

If $R^0 \geq R$, then $\Pi^*(P, R^0) \geq \Pi^*(P,R)$.

(4) $\Pi^*(P,R)$ is homogenous of degree one in the output and input prices.

$\Pi^*(tP, tR) = t\,\Pi^*(P, R)$ Where $t > 0$

(5) $\Pi^*(P,R)$ is convex in input and output prices.

$\Pi^*(tp + (1-t)P^0, tR + (1-t)R^0) < t\,\Pi^*(P, R) + (1-t)\,\Pi^*(P,R)$.

Convexity of the indirect profit function requires that

$$W' \frac{d^2\pi^*(P,R)}{d(P,R)^2} W \geq 0$$

(6) $D_2\Pi^*(P, R)$ is symmetric.

$$D_2\Pi^*(P,R) = \begin{bmatrix} \frac{d^2\pi^*}{dR^2} & \frac{d^2\pi^*}{dRdP} \\ \frac{d^2\pi^*}{dPdR} & \frac{d^2\pi^*}{dP^2} \end{bmatrix} = \begin{bmatrix} \frac{-dX^*(P,R)}{dR} & \frac{dX^*(P,R)}{dP} \\ \frac{dY^*(P,R)}{dR} & \frac{dY^*}{dP} \end{bmatrix} \ldots(17)$$

General Form:

$$D_2\Pi^*(P,R) = \begin{bmatrix} A & C \\ B & D \end{bmatrix}$$

Matrix A and Matrix D are both symmetric, and B = C.

$D_2\Pi^*(P, R)$ is symmetric and positive semi-definite. A necessary condition for positive semi-definiteness is that the main diagonal of $D_2\Pi^*(P, R)$ is greater than or equal to zero, i.e.

$$\frac{-dx^*_i}{dr_i} \geq 0 \qquad i = 1,2,\ldots\ldots,m \qquad \ldots\ldots\ldots(18)$$

$$\frac{dy^*_j}{dp_j} \geq 0 \qquad j = 1,2,\ldots\ldots,n \qquad \ldots\ldots\ldots(19)$$

Relation (21) can be written as:

$$\frac{dx^*_i}{dr_i} \leq 0 \qquad \ldots\ldots\ldots(20)$$

Relations (18) and (19) show that the supply function is upward sloping and the factor demand function is downward sloping.

Symmetry property of $D_2 \pi^*$ (P, R) implies the following:

(1) $\frac{dx^*_i}{dr_j} = \frac{dx^*_j}{dr_i}$ for all i' s and j' s.

(2) $\frac{dy^*_i}{dp_j} = \frac{dy^*_j}{dp_i}$ for all i' s and j' s.

(3) $\frac{-dx^*_i}{dp_j} = \frac{dy^*_j}{dr_i}$ for all i' s and j' s.

IV- THE PROPOSED MODEL

The indirect profit function can be represented by various functional forms. The form used in this paper is a translog function. because it has been described as flexible due to the fact that it has a sufficient number of parameters to allow for measurements of all relevant relationships without any prior restrictions (Hanoch, 1975). The translog form of the indirect profit function (14) can be written in a matrix notation as:

$$\bar{\pi} = \propto_0 + \propto \bar{d} + \frac{1}{2}\bar{d}\,'B\bar{d} \qquad \ldots..(21)$$

Where

$\bar{\pi} = ln\,\pi$, $\pi = PY' - RX' - F$, $\bar{d} = [\bar{P} \;\; \bar{R} \;\; \bar{Z}]$, $p = lnP$, $\bar{R} = lnR$,

$$\bar{Z} = [\,ln\,T' \quad \ln F\,]$$

T is defined as a time trend which is used as a proxy for technology and F represents the fixed cost.

The translog indirect profit function is expressed in a regular notation as

$$\ln \pi^*(\ln P, \ln R; \ln T, \ln F) = \propto_0 + \textstyle\sum_i \propto_i lnr_i + \sum_k B_k\, lnp_k + \frac{1}{2}\sum_i \sum_j \partial_{ij}\, lnr_i\, lnr_j$$

$$+\frac{1}{2}\sum_i \sum_k S_{ik}\, lnr_i\, lnr_{pk} + \frac{1}{2}\sum_i \sum_k d_{ik}\, lnp_i\, lnp_k + \sum_i b_i\, lnT\, lnr_i + \sum_i c_i\, lnT\, lnp_i + \sum_i h_i\, lnF\, lnr_i + \sum_k g_k\, lnF\, lnp_k \quad \ldots\ldots\ldots(22)$$

Using Hotelling Lemma, the first partial derivatives of (22) with respect to lnP and lnR are equal to revenue(myk) and expense shares(mxi) of profit; i.e.

$$\frac{dln\pi^*}{dlnr_i} = \frac{d\pi^*}{dr_i}\frac{r_i}{\pi^*} = -x_i\frac{r_i}{\pi^*} = -M_{xi}$$

$$-M_{xi} = \alpha_i + \sum_j \sigma_{ij}\, lnr_j + \sum_k S_{ik}\, lnP_k + b_i\, lnT + h_i\, lnF \quad \ldots\ldots(23)$$

$$\frac{dln\pi^*}{dlnp_k} = \frac{d\pi^*}{dp_k}\frac{p_k}{\pi^*} = y^*_k\frac{p_k}{\pi^*} = M_{yk} = B_k + \sum_i S_{ki}\, lnr_i + \sum_i d_{ik}\, lnP_i + c_i\, lnT + g_k\, lnF \quad \ldots\ldots(24)$$

V. Theoretical implications of the model:

(1) The linear homogeniety of the indirect profit function requires:

which requires the following to be held simultaneously:

(a) $\sum_i \alpha_i + \sum_i B_K = 1$

(b) $\sum_i \delta_{ij} + \sum_k S_{ik} = 0$

(c) $\sum_i b_i + \sum_i C_i = 0$

(d) $\sum_i S_{Ij} + \sum_i d_{Ik} = 0$

The linear homogeneity of the indirect profit function implies that the share equations are homogenous of degree zero in all prices.

(2) The symmetry property of the $D_2\, \pi^*(P,R)$ requires the following to be true:

$S_{ij} = S_{ji}$

$d_{ij} = d_{ji}$

(3)The price elasticities of choice (weaver: 1983)

$$\varepsilon_{ij} = \frac{dx_i}{dr_j}\frac{r_j}{x_i} = -\delta_{ij}\, M_{XJ} - \frac{1}{M_{XI}}$$

$$\varepsilon_{ii} = \frac{dx_i}{dr_j}\ \frac{r_j}{x_i} = -1 - \frac{\delta II}{M_{XI}} - M_{xi}$$

$$\mu_{ik} = \frac{X_i}{P_k}\ \frac{pk}{x_i} = K_{yk} - \frac{S_{ik}}{M_{xi}}$$

$$e_{ik} = \frac{Y_i}{P_k}\ \frac{P_K}{y_i} = M_{yk} + \frac{d_{ik}}{M_{yl}}$$

$$e_{kk} = -1 + M_{YK}\ \frac{d_{kk}}{M_{yx}} = \frac{\delta_{yr}}{\delta_{pr}}\ \frac{P_r}{Y_r}$$

$$E_{lj} = \frac{Y_l}{r_j}\ \frac{r_j}{Y_L} = \frac{S_{LJ}}{M_{YL}} - M_{xj}$$

(4) Return to size (PTSZ) (weaver: 1983).

RTSZ can be measured with parameters of the indirect profit function as follows:

$$RTSZ = \sum_{j=1} = \frac{rj}{\pi^*}\ \frac{d\pi}{drj} \Big/ \sum_k = \frac{p_k}{\pi^*}\ \frac{d\pi}{dpi}$$

$$RTSZ = \sum_{j=1} M_{xj} \Big/ \sum_k M_{YK}$$

If RTSZ = l; constant return to Size.
If RTSZ > l; increasing return to size,
If RTSZ < l; decreasing return to size
(5) Measure of technological change.
Hicks definition of technological change is:

$$X_i \left.\begin{matrix}\text{Saving}\\ \text{Neutral}\\ \text{Using}\end{matrix}\right\} \text{Relative to } x_j \text{ if } K_{ij} \gtreqless 0$$

Where $K_{ij} = d\left(\frac{x_i}{x_j}\right) / dt$

"Given the input & output price are constant"

In this study technological change will be measured by using the estimated parameters of the estimated model in the following way:

Technological change is biased toward saving x_i and using x_i if $M_{ij} \gtrless 0$, where $M_{ij} = b_i M_{xj} - b_j M_{xi}$ ....(25)

(using / saving)

(6) Economic Relationships:

If $\frac{dx_i}{dr_j} \geq 0$, then input i and j are substitutes.

If $\frac{dx_i}{dr_j} \leq 0$, then input i and j are complements.

If $\frac{dy_l}{dp_k} \geq 0$, then output l and k are complements.

If $\frac{dy_1}{dp_k} \leq 0$, then output land k are competitive.

VI. STATISTICAL, EVALUATION OF THE MODEL

a. The model to be estimated is written in the following form:

$$M_{Y1} = B_1 + S_{11} Lnr_1 + S_{12} Lnr_2 + S_{13} Lnr_3 + S_{14} Lnr_4 + d_{11} LnP_1 + d_{12} LnP_2 + C_1 LnT_1 + Y_1 LnF + e_1$$

$$M_{Y2} = B_2 + S_{21} Lnr_1 + S_{22} Lnr_2 + S_{23} Lnr_3 + S_{24} Lnr_4 + d_{21} LnP_1 + d_{22} LnP_2 + C_2 LnT_2 + Y_2 LnF + e_2$$

$$M_{x1} = -\alpha_1 - \delta_{11} Lnr_1 - \delta_{12} Lnr_2 - \delta_{13} Lnr_3 - \delta_{14} Lnr_4 - S_{11} LnP_1 - S_{12} LnP_2 - b_1 LnT - h_1 LnF - e_3$$

$$M_{x2} = -\alpha_2 - \delta_{21} Lnr_1 - \delta_{22} Lnr_2 - \delta_{23} Lnr_3 - \delta_{24} Lnr_n - S_{21} LnP_1 - S_{12} LnP_1 - S_{22} LnP_2 - b_2 LnT - h_2 LnF + e_4$$

$$M_{x3} = -\alpha_3 - \delta_{31} Lnr_1 - \delta_{32} Lnr_2 - \delta_{33} Lnr_3 - \delta_{34} Lnr_4 - S_{31} LnP_1 - S_{32} LnP_2 - S_{32} LnP_2 - b_3 LnT - h_3 LnF + e_5$$

$$M_{x4} = -\alpha_4 - \delta_{41} Lnr_1 - \delta_{42} Lnr_2 - \delta_{43} Lnr_3 - \delta_{44} Lnr_4 - S_{41} LnP_1 - S_{42} LnP_2 - b_4 LnT - h_4 LnF + e_6$$

Where:

M_{yi} = Profit share of output i, i = 1, 2.

M_{xj} = Profit share of input j, j = 1, 2, 3, 4.

r_j = price paid by farmer for input j (index number)

p_i = Price received by farmer for output i (index number)

T = Time trend.

$F^{(1)}$ = Discounted cost of miscellaneous expenses.

Y_1 = Livestock and livestock product.

Y_2 =All crops.

x_1 = Feed, x_2 = fertilizer.

x_3 = Seed. x_4 = farm labor.

Regression analysis of single equation ordinary least square estimates was applied on the different share equations.

The estimated parameters are reported in the following tables:

Table 1

Parameter Estimates for the Input Profit Share Equations.

Share Equation	Parameter	Value	Std. Dev.	T-Ratio
M_{y1} $R^2 = 74.68$	B_1	-114.925	----	----
	d_{11}	-13.394	10.7	-1.3
	d_{22}	-2.522	18.6	0.14
	S_{11}	7.656	17.2	0.4
	S_{12}	6.404	8.5	0.75
	S_{13}	-24.051	16.6	-1.4
	S_{14}	32.839	17.2	1.9
	g_1	16.339	18.9	0.86
	C_1	-2.786	2.9	0.95
M_{y2} $R^2 = 76.67$	B_2	-106.2	----	----
	d_{12}	-12.9	10.9	-1.2
	d_{22}	-3.8	19.1	-0.2
	S_{21}	12.3	17.7	0.7
	S_{22}	5.3	8.7	0.6
	S_{23}	-23.0	17.1	-1.3
	S_{24}	32.2	17.6	1.8
	g_2	11.89	19.5	0.6
	C_2	-2.8	3.0	-0.9

1 - Miscellaneous expenses are treated as fixed cost.

Table 2
Parameter Estimates for the Input Profit Share Equations.

Share Equation	Parameter	Value	Std. Dev.	T-Ratio	R^2
M_{x1}	σ_1	30	----	----	0.748
	S_{11}	4.3	2.9	1.4	
	S_{12}	1.6	5.2	0.3	
	δ_{11}	-4.3	4.8	-0.9	
	δ_{12}	-1.4	2.4	-0.6	
	δ_{13}	6.8	4.7	1.5	
	δ_{14}	-9.3	4.8	-1.9	
	Z_1	-3.9	5.3	-0.7	
	b_1	-0.75	0.82	-0.9	
M_{x2}	σ_2	-2.9	----	----	0.550
	S_{21}	0.228	2.1	0.1	
	S_{22}	0.5	3.6	0.13	
	δ_{21}	-0.16	3.4	-0.05	
	δ_{22}	-0.002	1.7	-0.001	
	δ_{23}	1.4	3.2	0.4	
	δ_{24}	-3.9	3.3	-1.2	
	Z_2	2.1	3.7	0.6	
	b_2	0.4	0.6	0.6	
M_{x3}	σ_3	2.2	----	----	0.68
	S_{31}	0.4	0.8	0.5	
	S_{32}	0.3	1.4	0.2	
	δ_{31}	-0.4	1.3	-0.3	
	δ_{32}	0.05	0.6	0.08	
	δ_{33}	0.63	1.3	0.5	
	δ_{34}	-1.9	1.3	-1.4	
	Z_3	0.3	1.4	0.2	
	B_3	0.2	0.2	0.95	
M_{x4}	σ_4	28.9	----	----	0.80
	S_{41}	3.5	1.5	2.3	
	S_{42}	0.7	2.6	0.3	

	δ_{41}	-2.7	2.4	-1.1	
	δ_{42}	-1.6	1.2	-1.3	
	δ_{43}	4.9	2.3	2.1	
	δ_{44}	-5.4	2.4	-2.2	
	Z_4	-4.9	2.7	-1.9	
	b_4	0.43	0.42	1.1	

b. Estimating own- and cross-price elasticities of outputs and inputs. Table 3 shows these estimated elasticities.

Table 3
Means [1] of price elasticities of choice (1966-1980).

Prices of Input / output Output Input	y_1	y_2	x_1	x_2	x_3	x_4
y_1	1.4	1.7	0.25	1	-6.9	5.9
y_2	4.3	2.4	1.6	0.6	-5.5	6.7
x_1	1.5	4.12	-3.0	0.07	-2.1	5.7
x_2	1.6	3.5	-1.45	-1.6	-1.9	1.1
x_3	2.9	2.8	-4.48	5.03	-4.35	-3.62
x_4	-0.1	3.3	2.08	-0.46	-0.44	-0.01

c. Estimated return to size is calculated using the mean of the inputs and outputs profit shares. The result indicated that RTSZ = 0.3, which implies that there is decreasing return to size. This simply means that profit cannot be increased by further size expansion.

d. Technological change is calculated according to equation (32). Results are shown in Table 4.

Table 4
Measure (Mean) of Techno logical change

Inputs	Fertilizer	Seed	Labor
Feed	-0.97	-0.11	-0.03
Fertilizer		-0.04	0.022
Seed			0.054

(1)- Means is based on taking the avg value of input and output share, respectively.

VII- Statistical Evaluation of The Model

A: tests for multi-colinearitys

Results of the estimated model showed the existence of multi-co linearity among the explanatory variable . This problem led to some difficulties of the precision of the estimated parameters and very large sampling variance of the estimated coefficients, thus It is almost impossible to make any hypothesis testing in this case (Johnston.1960), Correction for this phenomena was carried out.

B: Test for Symmetry

The indirect profit function is symmetric if the matrix B in equation (26) is symmetric .

The symmetry of matrix B implies that $\delta_{ij} = \delta_{ji}$

$d_{lk} = d_{kl}$ I & j =1,2,3,4

$S_{ij} = S_{ji}$ k,1=1,2

$S_{il} = S_{li}$

Let $H_0 : \delta_{ij} = \delta_{ji}$

$H_A : \delta_{ij} \neq \delta_{ji}$

$$t = \frac{\delta_{ij} - \delta_{ji}}{\sigma(\delta_{ij})} \sqrt{n}$$

n = sample size.

σ (δ_{ij}) = the std. dev. of coefficient (δ_{ij})

If $|t^*| > |t_c|$ reject H_0.

$|t^*| \leq |t_c|$ fail to reject H_0.

Where t_c = critical t value

VIII. SUMMARY

In this paper it is demonstrated how one can derive and estimate share equations from the indirect profit function. This method is very convenient when there is no data available for the quantities of outputs produced and the quantities of inputs used in the production process.

It is obvious that from the output and input share equations that the supply and factor demand functions could be obtained. In general, we need to take the antilog of the indirect profit function and then apply Hotelling Lemma to get the supply and factor demand functions.

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