Measurement of Level Density Using Spectral Distribution Method and Comparison with Bethe Formula

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Abstract:

Nuclear level densities of some heavy nuclei have been calculated taking into consideration the effect of one-body interaction. We use the Hamiltonian moments method based upon the statistical calculation using the Nilsson Hamiltonian for the single particle energies. The density is then constructed in terms of the moments of the Hamiltonian which are defined using spectral distribution methods. The results indicated that the Gaussian function is appropriate to describe the nuclear level density. The present calculated results have been compared with the standard Bethe formula and good agreement was obtained. Bethe parameter has been determined during the fit.

Keyword: Level density, Nilsson model, Bethe formula, Hamiltonian.

الملخص:

في هذا العمل تناولنا بالدراسة الكثافة النووية لمستويات طاقة الأنوية وقمنا باستخدام القوانين الخاصة بنظرية التوزيع الطيفي مع الأخذ بعين الاعتبار أن الجسيمات داخل النواة (البروتونات والنيوترونات) غير متفاعلة مع بعضها البعض. ثم المقارنة بين النتائج المتحصل عليها لعدد من الأنوية الثقيلة وبين صيغة بيثية باعتبارها المعادلة المتفق عليها لحساب كثافة المستويات النووية وتوصلنا إلى نتائج جيدة خلال هذه المقارنة.

الكلمات الرئيسية: كثافة المستوى، نموذج نيلسون، صيغة بيثية، هاميلتونيان.

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Introduction

Nuclear level density is one of key ingredients of the nucleus (1),(2),(3) and plays a crucial role in both pure and applied physics. This importance comes from the wide needs for understanding the nuclear system properties such as the description of excited nuclei, the fission dynamics and the calculation of reaction cross sections.

Given the experimentally observed spectrum of a nucleus, level density is simply defined as the number of excited levels in a given energy interval.

Theoretically, the calculation of level density starts by defining the Hamiltonian (4) of the system and calculating the eigenvalues and eigenfunctions. In the rough approximation, the calculation of level density is traditionally based (5),(6),(7) on treating the nucleus as a group of non-interacting Fermions. The non-interacting Fermi gas model of particles moving in orbits independently of each other has led to various expressions for the density. In particular the Bethe level density formula (8),(9) which is presented as a function of the excitation energy such as:

$$\rho(E) = \rho_0 \exp(2\sqrt{aE}) \ldots\ldots\ldots\ldots\ldots (1)$$

where a is the Bethe parameter and p_0 is the density at ground state energy taken at 0 MeV.

In the current work we use the spectral distribution method of French (10),(11) for the calculation of level density. The basis of this method is to determine the level density not from the nuclear spectrum itself but from the level distribution defined in terms of the moments of the Hamiltonian. We describe the calculation of nuclear level density assuming that the nucleons do not interact with one another but move independently in a mean field created by a single particle Hamiltonian. We first define the Hamiltonian in terms of the single particle energies and then calculate the moments of the given spectroscopic space. The non-interacting particle (NIP) level density is then constructed in terms of three low-order moments. In terms of the dimensionality, centroid and variance of the distribution, the density approximated as a Gaussian function may be written as follow:

$$\rho(m,E) = \frac{d(m)}{\sqrt{2\pi\sigma^2(m)}} \exp\left(-\frac{(E-E_c(m))^2}{2\sigma^2(m)}\right) \ldots\ldots\ldots\ldots\ldots (2)$$

Where $d(m), E_c, \sigma^2(m)$ the dimensionality, centroid and variance of the distribution respec*tively.*

We only content by presenting the results of this work. For additional details regarding how the calculations are carried out, we recommend referring to the references (10),(11),(12) for full statistical information.

Results and Discussion

We determine the non-interacting particle (NIP) level density for nuclei. For the calculations we first have to define the single particles orbits energies using the Nilsson model (13) with zero-deformation where the active protons and neutrons will be distributed. By looking at the energies of these orbits and keeping in mind the maximum excitation energy of interest, we define a core nucleus in which all the orbits are completely filled. The core nucleus is treated as inert meanwhile the remaining protons and neutrons define the valence nucleons.

Let us take ^{172}Yb as an example:

$$^{172}_{70}Yb \quad ^{148}_{58}Ce + 12p + 12n \ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

We take ^{148}Ce as the core nucleus for ^{172}Yb, therefore, we are left with 12 active protons and 12 active neutrons. The choice of the active orbits depends on their energy values. They should be close enough for excitations so as not to exceed the maximum excitation energy of interest. We have chosen four active orbits for protons ($d_{\frac{5}{2}}, d_{\frac{11}{2}}, d_{\frac{3}{2}}, s_{\frac{1}{2}}$) and five active orbits for neutrons ($d_{\frac{9}{2}}, i_{\frac{13}{2}}, p_{\frac{3}{2}}, f_{\frac{5}{2}}, p_{\frac{1}{2}}$)..

Tables (1) and (2) show the single particle orbits for protons and neutrons respectively.

We calculated the level density for several heavy nuclei assuming that the nucleons are non-interacting particles and these particular nuclei have been chosen because of the availability of experimental data (14).

Next we fit the calculated density to the one parameter Bethe formula, this test allows us to evaluate how close the calculated data from the original Bethe expression.

Table (3) presents the fitting of nuclear level density results with Bethe formula.

Table (1): ^{172}Yb proton single particle energies (SPE) as functions of Nlj quantum numbers. The first 12 orbits are occupied by the protons of the core nucleus ^{148}Ce.

N	l	j	SPE (MeV)
0	0	0.5	11.058
1	1	1.5	17.962
1	1	0.5	19.366
2	2	2.5	24.584
2	2	1.5	26.925
2	0	0.5	27.21
3	3	3.5	30.925
3	3	2.5	34.202
3	1	1.5	34.677
3	1	0.5	36.081
4	4	4.5	36.984
4	4	3.5	41.197
4	2	2.5	41.863
5	5	5.5	42.762
4	2	1.5	44.203
4	0	0.5	44.488
5	5	4.5	47.911
5	3	3.5	48.767
5	3	2.5	52.043
5	1	1.5	52.519
5	1	0.5	53.923

Table (2): ^{172}Yb neutron single particle energies (SPE) as functions of Nlj quantum numbers. The first 17 orbits are occupied by the protons of the core nucleus ^{148}Ce.

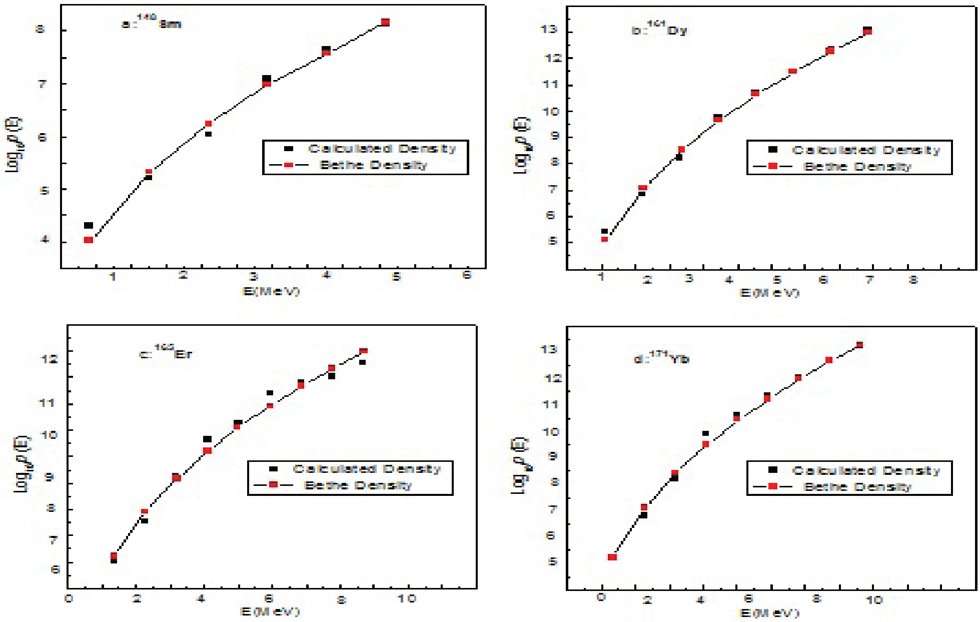
N	l	j	SPE (MeV)
0	0	0.5	11.058
1	1	1.5	17.962
1	1	0.5	19.367
2	2	2.5	24.668
2	0	0.5	26.789
2	2	1.5	27.009
3	3	3.5	31.177
3	1	1.5	34.088
3	3	2.5	34.454
3	1	0.5	35.493
4	4	4.5	37.489
4	2	2.5	41.189
4	4	3.5	41.701
4	0	0.5	43.311
4	2	1.5	43.53
5	5	5.5	43.603
5	3	3.5	48.094
5	5	4.5	48.752
6	6	6.5	49.519
5	1	1.5	51.005
5	3	2.5	51.370
5	1	0.5	52.409
6	4	4.5	54.8
6	6	5.5	55.604
6	2	2.5	58.50
6	4	3.5	59.013
6	2	1.5	60.842
6	0	0.5	60.623

Table 3: The results of the fitting between the NIP density and Bethe formula

Nucleus	Bethe Parameter(a)
^{148}Sm	12.089
^{149}Sm	12.941
^{160}Dy	15.767
^{161}Dy	16.285
^{162}Dy	16.379
^{165}Er	8.369
^{167}Er	8.99
^{170}Yb	13.157
^{171}Yb	11
^{172}Yb	8.734

We also show in the next figures, the comparison of some chosen nuclei, between the corresponding calculated NIP density and the Bethe expression.

From the close agreement that found, it is clear that the Bethe formula is a good approximation to the calculated NIP density, this observation is not surprising since Bethe formalism treats the nucleons as non-interacting Fermi gas particles.



the Bethe formula. The black squares represent the calculated density meanwhile the Bethe density is indicated by the red squares.

Conclusion - future steps

In this part of the work, we have dealt with one of the oldest and important topics in nuclear physics-the calculation of nuclear level densities. The spectral distribution methods developed for studying quantities of general interest in nuclear structure enable us to go beyond and calculate nuclear level density with and without the interactions between nucleons.

The initial calculation was based upon the low-order moments (dimensionality, centroid and variance) of the Hamiltonian and for these, the results indicated that, the Gaussian function is appropriate to describe the level density.

By fitting the calculated density to the standard Bethe formula, we were also able to determine the value of the Bethe parameter a. It is clear that the Bethe formula is a good approximation to the calculated non-interacting particle density.

In future, we recommend that, the calculation should move from the effect of one body Hamiltonian to one plus two body effects. Therefore, it is important to make detailed comparisons of such calculations with data and with the results of calculations including the full two-body force. The corresponding interacting particle (IP) density should be calculated for several nuclei where the desired information exists and compared to experimentally measured data.

Our calculations were done with the Nilsson zero-deformation single particle energies. It would be worthwhile to study the effects on the density of varying the single particle energies as a function of the deformation parameter.

Also, the method has been tested in the rare-earth mass region (148 $\geq$A$\leq$172). In the future this calculation should be extended to other regions such as light and heavier nuclei where the experimental data are available.

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