An Optimized Image Watermarking Technique Based on LU Factorization and Entropy Analysis

■ Omar Abodena *

● Received: 03/ 02 / 2024 ● Accepted : 22/ 05 /2024

■ **Abstract:**

An optimized image watermarking technique based on LU factorization and entropy analysis in combination with lifting wavelet transform and discrete cosine transform is presented in this paper. At first, the original image is decomposed by a 2-level lifting wavelet transform for obtaining the coefficients of a high-frequency subband followed by discrete cosine transform. Afterward, non-overlapping blocks are obtained by dividing the coefficients of discrete cosine transform whereas LU factorization is applied to each non-overlapping blocks based on pseudo-random sequences. Then, the watermark is embedded into the first row, the first column element of the upper triangular matrix of LU factorization. The normalized cross-correlation (NC), and the peak signal-to-noise ratio (PSNR) are used to evaluate the invisibility and robustness of the presented technique. The experimental results have indicated that the presented technique fulfills all watermarking requirements in terms of invisibility, robustness, security, and capacity. The comparison with the existing scheme has shown that the proposed watermarking technique has a superior performance in terms of invisibility than the existing scheme.

● **Keywords:** Watermarking, Discrete cosine transform, Entropy analysis, Lifting wavelet transform, LU factorization

■ **المستخلص:**

تم في هذه الورقة تقديم تقنية محسنة للعلامة المائية للصور تعتمد على تحليل الإنتروبيا إلى عوامل LU بالإضافة إلى تحويل المويجات الرفعية وتحويل جيب التمام المنفصل. في البداية، يتم

* PhD, Computer Science Dept., Faculty of Science Gharyan, University of Gharyan,
Email: omar.abodena@gu.edu.ly

تحليل الصورة الأصلية عن طريق تحويل مويجات الرفع ذات المستويين للحصول على معاملات النطاق الفرعي عالي التردد متبوعًا بتحويل جيب التمام المنفصل. بعد ذلك، يتم الحصول على الكتل غير المتداخلة عن طريق قسمة معاملات تحويل جيب التمام المنفصل بينما يتم تطبيق عامل LU على كل الكتل غير المتداخلة بناءً على تسلسلات شبه عشوائية. بعد ذلك، يتم تضمين العلامة المائية في الصف الأول، وعنصر العمود الأول للمصفوفة المثلثية العليا لتحليلLU. تم استخدام الارتباط المتبادل الطبيعي، ونسبة الذروة للإشارة إلى الضوضاء لتقييم اختفاء ومتانة التقنية المقدمة. أشارت النتائج التجريبية إلى أن التقنية المقدمة تلبي جميع متطلبات العلامة المائية من حيث الاختفاء والمتانة والأمن والسعة. أظهرت المقارنة مع التقنية المحددة أن تقنية العلامة المائية المقترحة تتمتع بأداء متفوق من حيث الاختفاء عن التقنية المحددة.

● **الكلمات المفتاحية:** تحليل LU، تحليل الانتروبيا ، تحويل جيب التمام المنفصل ، رفع تحويل المويجات ، العلامة المائية

1. Introduction

With the explosive development of the network, multimedia information (images, videos, and audio) today become the most utilized form. Digital multimedia productions entice the attention of security experts with the appearance of electronic appliances and the needed cloud computing. The digital watermarking technique has been considered an effective tool for copyright protection of digital multimedia productions, in which the information hidden in multimedia can withstand different attacks without degrading its quality (1). In addition, digital watermarking techniques have been utilized in various applications like tamper detection, fingerprinting, broadcast monitoring, content authentication, and content identification. The watermarking technique properties can be roughly divided into four categories: imperceptibility, robustness, security, and watermark capacity, it is a big issue to fulfill all properties of a watermarking technique together (2).

Over the last decade, several techniques have been presented to fulfill the watermarking characteristics which can be categorized into domains: spatial domain (3, 4) and transform domain including discrete cosine transform

(DCT) (5, 6), discrete Fourier transform (7-9), lifting wavelet transform (LWT) (10, 11), Walsh Hadamard transform (12, 13), and matrix factorization like singular value decomposition (SVD) (14, 15), Hessenberg factorization (16, 17), QR factorization (18, 19), LU factorization (20, 21), and Schur factorization (22, 23).

In this paper, an optimized image watermarking technique based on LU factorization and entropy analysis in combination with lifting wavelet transform, and discrete cosine transform is proposed for copyright protection. At first, the original image is decomposed by a 2-level LWT for obtaining the coefficients of a high-frequency subband followed by DCT. Afterward, non-overlapping blocks are obtained by dividing the coefficients of DCT whereas LU factorization is applied to each non-overlapping blocks based on pseudo-random sequences. Then, the watermark bits are hidden in the elements of the first row, the first column of the upper triangular matrix of LU factorization.

The main contributions of the work are summarized as follows:

1. An effective watermarking technique based on the combination of DCT and LWT with LU factorization is presented for grayscale image watermarking. In this work, the benefits of hybridization of this combination are used to enhance invisibility and robustness against attacks, such as LWT provides excellent frequency localization characteristics, LU factorization as matrices factorization offers various elements that be utilized for embedding the hidden information (watermark) into the highest element value of the upper triangular matrix of LU factorization, and DCT provides energy compaction characteristics.
2. A random permutation function is utilized for the generation of a pseudo-random sequence to improve the security of the presented technique by selecting the blocks based on the pseudo-random sequence for hidden information bits (watermark).
3. To derive the optimum quantization step, the entropy analysis is utilized to have the best quantization step for embedding the watermark. The

presented grayscale image watermarking method can achieve a better trade-off between invisibility and robustness.

The rest of this paper is categorized as follows. In Section 2, the various terminologies such as LWT, DCT, LU factorization, entropy analysis, and random permutation function are explained. The discussions of the presented watermarking method are given in Section 3. The experimental results and compared schemes are provided in Section 4. The conclusion is given in Section 5.

2. Preliminaries

2.1. Lifting wavelet transform

LWT is presented to reduce aliasing effects and distortion and requires less memory. LWT can be used as the best choice to have a lower computational complexity than traditional DWT (24). The lifting wavelet transform can be used in several applications like image watermarking, compression, image processing, and pattern recognition as a robust technique. Lifting wavelet transform requires half the number of calculations in comparison to ordinary DWT and lifting wavelet transform allows us to do reversible integer wavelet transform (25). Three steps of LWT are below:

1) Splitting: Dividing a signal $sg(l)$ into even and odd non-overlapping patterns: $Sgo(l)$ and $Sge(l)$.

$$Sgo(l) = sg(2l + 1), Sge(l) = sg(2l) \quad (1)$$

2) Prediction: Predicting odd signal patterns based on even signal patterns and the even signal and odd signal should be widely correlated. As a result of this step, viewed a high-pass filter processing. The result of a difference between the prediction step and the splitting step is below:

$$D(l) = Sgo(l) - P(Sge(l)) \quad (2)$$

where $D(l)$ indicates as the high-frequency component of the signal $sg(l)$, whereas $P[.]$ indicates as the predict operator.

3) Updating: Introducing the updating operator U [.], and using the detail signal $D(l)$ for updating even samples $Sge(l)$. This step can be viewed as a low-pass filter processing. The approximate signal $C(l)$ can be given as follows:

$$C(l) = Sge(l) + U(D(l)) \quad (3)$$

where the $C(l)$ denotes the low-frequency component of the signal $sg(l)$.

2.2. Discrete cosine transform

DCT provides an orthogonal decomposition of a real number and has to calculate the images as part of a complex number. Discrete cosine transform is given by extracted items with cosine in DFT. A transformation matrix of DCT provides a symmetrical coefficient matrix with the same summation between every auxiliary diagonal that is parallel to a diagonal. DCT has several benefits including high computational complexity, high compression ratio, and low error rate (26).

2.3. LU factorization

LU factorization is a procedure of factorization of a matrix into a lower-triangular matrix and an upper-triangular matrix. A real matrix A is decomposed by the LU factorization into *L* and *U* matrices, and its equation can be given as follows (20):

$$A = LU \quad (4)$$

where *L* is a lower triangular matrix, whilst *U* is an upper triangular matrix.

2.4. Entropy analysis

The entropy analysis has been utilized for listing how much information content is given in the image. The more contained image data denotes good quality. It represents the number of randomness and uncertainty included in the image. Also, entropy analysis is called Shannos's entropy, and the data entropy computes the uncertainty of a data source. The entropy of Shannon can be defined as follows (25, 27){Kumar, 2021 #29}:

$$E(x) = -\sum p(x) \log p(x) \quad (5)$$

3. Proposed Scheme

A new grayscale image watermarking technique is presented for hiding the watermark bits in an image based on the combination of LWT and DCT with LU factorization, and entropy analysis. The presented technique has two steps: the watermark embedding step and the watermark extraction step as shown in figures 1 and 2 that indicate the block diagrams of the steps.

3.1. Watermark embedding step

The watermark embedding step is explained below:

Step 1: Performing a 2-level lifting wavelet transform to the original image of size $N \times N$ for transformations. To have cA_1, cH_1, cV_1, and cD_1 of the 1-level followed by cA_2, cH_2, cV_2, and cD_2 of the cD_1 of the 2-level.

Step 2: The DCT is performed on the cD_2 high-frequency sub-band to have DCT coefficients.

Step 3: The DCT coefficients are divided for having non-overlapping blocks of size 4 × 4. In which the blocks' number is 64^2

Step 4: The entropy of each block is computed to get the average of the blocks' entropy which be used as quantization stage α.

Step 5: The pseudo-random sequences are generated by randperm-function based on the key_1 and key_2 to obtain two random numbers that indicate the row and column.

Step 6: The LU factorization is performed on selected blocks of 4 × 4 utilizing the pseudo-random sequences as mentioned in Eq. (4) to get the matrices L and U.

Step 7: The magnitudes M_1 and M_2 are modified depending on the binary watermark data w, as below:

$$if\ w(i,j) = 1, \begin{cases} M_1 = 0.25\alpha \\ M_2 = -0.50\alpha \end{cases} \quad (6)$$

$$if\ w(i,j) = 0, \begin{cases} M_1 = -0.25\alpha \\ M_2 = 0.50\alpha \end{cases} \quad (7)$$

here α denotes the quantization stage, and $(1 \leq i, j \leq N)$.

Step 8: The potential quantization outcomes T_1 and T_2 are calculated based on the modified magnitudes M_1 and M_2.

$$T_1 = 2k\alpha + M_1 \quad (8)$$

$$T_2 = 2k\alpha + M_2 \quad (9)$$

where $k = floor(ceil(u_{1,1}/\alpha)/2)$, $ceil(.)$ and $floor(.)$ denotes as the largest nearest integer and the least nearest integer, respectively.

Step 9: selection the first-row, first-column element $u_{1,1}$ of upper triangular matrix U for hidden the binary watermark as shown:

$$d'_{1,1} = \begin{cases} T_2 \; if \; abs(u_{1,1} - T_2) < abs\,(u_{1,1} - T_1) \\ T_1 \; else \end{cases} \quad (10)$$

here abs (.) indicates the absolute value.

Step 10: The $u_{1,1}$ with $u'_{1,1}$ is replaced and then perform the inverse LU factorization to have the A' watermarked block.

$$A' = LU' \quad (11)$$

Step 11:Repeating steps 6–10 to hide all watermark bits.

Step 12:Combination all watermarked blocks together.

Step 13:Performing inverse of DCT for combination watermarked blocks to have the altered high-frequency sub-band cD^w, in which cD^w denotes high-frequency sub-band estimation of the watermarked.

Step 14:The inverse of the 2-level LWT is performed based on the changed high-frequency sub-band cD^w instead of cD_2 for obtaining the watermarked image.

3.2. Watermark extraction process

The watermark embedding process is presented below:

Step 1: A 2-level LWT is applied to the watermarked image of size $N \times N$ for transformations. To have cA_1, cH_1, cV_1, and cD_1 of the first-level then having cA_2, cH_2, cV_2, and cD_2 of the cD_1 of the second-level.

Step 2: The DCT is performed on the cD_2 high-frequency subband to have coefficients of DCT.

Step 3: Dividing DCT coefficients for having non-overlapping blocks of size 4 × 4. Where the blocks' count is 64^2.

Step 4: Computing entropy of each block is for having the mean of the entropy blocks which be used as scaling factor α.

Step 5: The pseudo-random sequences are generated by randperm-function based on the key_1 and key_2 to obtain two random numbers that indicate the row and column.

Step 6: The LU factorization is performed on selected 4 × 4 blocks based on the pseudo-random sequences as mentioned in Eq. (4) to get the matrices L' and U'.

Step 7: Selection of the first-row, first-column element $u'_{1,1}$ of upper triangular matrix U' for extraction of the watermark bits as given:

$$w'_{i,j} = mod\ (\ ceil\left(\frac{u'_{1,1}}{\alpha}\right), 2) \qquad (12)$$

where $mod(.)$ denotes the modulo peration and $(1 \le i,\ j \le N)$.

Step 8:Obtaining the extracted watermark bits by repeating steps 6–7.

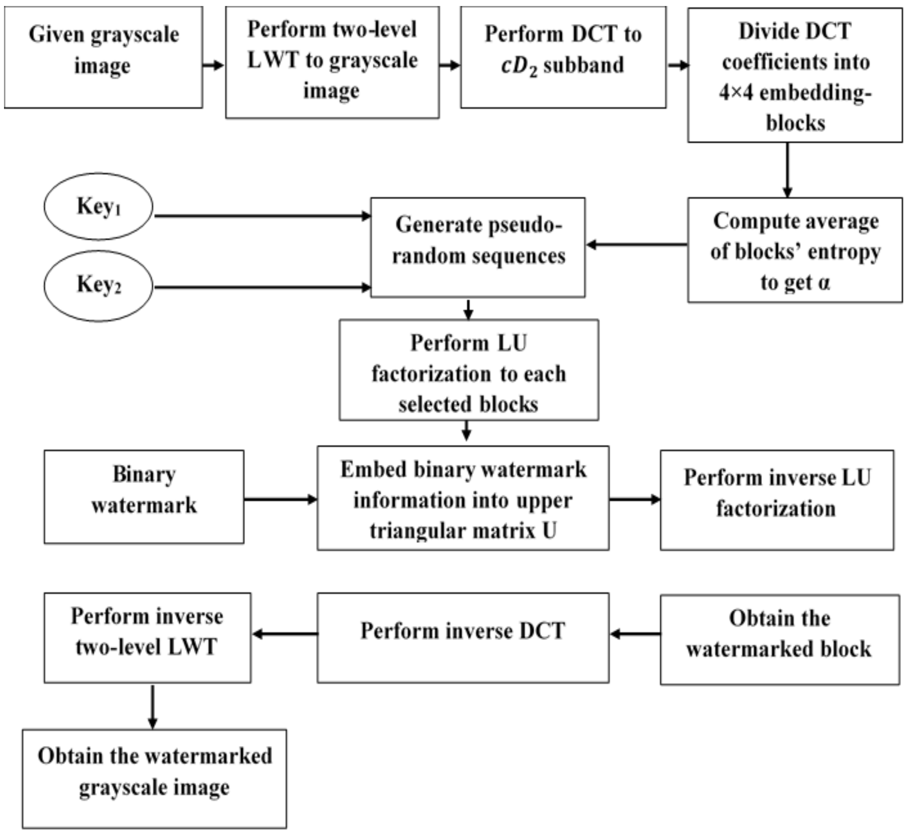

Figure 1: The proposed watermarking embedding step

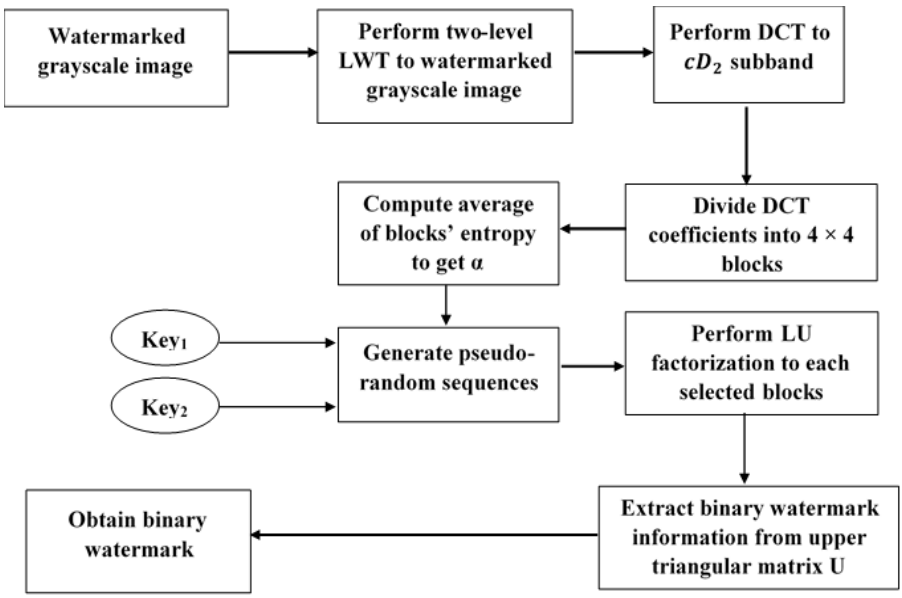

Figure 2: The proposed watermarking extraction step

4. Experimental results and discussion

For all experiments, the presented watermarking technique is performed on common test images. Five test images from the USC-SIPI (28) database are used in the presented watermarking technique, the size of the images is 1024 × 1024 as given in figure 3. The technique uses various watermarks of size 64×64 as given in figure 4. The experiment results were simulated utilizing MATLAB R2017b on a laptop whose classifications are Intel Core i3 2.00 GHz CPU, windows 10 environment, and 4.00 GB RAM.

The attacks performed in the presented technique are below:

· Flipping: Flipping the watermarked images up to down.

· Gamma correction: Applying the Gamma correction with a 1.0 Gamma value to the watermarked images.

· Sharpening: Sharpening the watermarked images with a 0.1 amount value.

· Blurring: Blurring with a 0.2 blurring value to the watermarked images.

· Poisson noise: Adding the Poisson noise to the watermarked images.

· Scaling: Scaling the watermarked images to 2 times.

· JPEG 2000 compression: Performing watermarked images by applying the JPEG 2000 compression with 1 compression ratio.

The normalized cross-correlation (NC), and the peak signal-to-noise ratio (PSNR) are used to evaluate the invisibility and robustness of the presented technique.

PSNR can be known as an engineering phrase for the calculation of the ratio between the highest potential power of a signal, and the power of distorting noise that affects the quality of the representation signal (29). PSNR equation is below:

$$PSNR = 10 \log \frac{255^2}{MSE} dB \tag{13}$$

where MSE denotes the average square error between the watermarked image and the cover image. MSE equation is below:

$$MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (o(i,j) - w'(i,j))^2 \tag{14}$$

here, $w'(i,j)$ and $o(i,j)$ denote the $(i,j)^{th}$ pixel value in the watermarked image, and cover image, respectively.

NC is a well-known metric used to measure the similarity level between a watermarked image and a cover image (30). NC equation is below:

$$NC = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} O(i,j) \times O'(i,j)}{\sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} O^2(i,j)} \times \sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} O'^2(i,j)}} \tag{15}$$

here, O' and O denote the extracted watermark image, and the original watermark image, respectively; *N* and *M* denote the size of watermark.

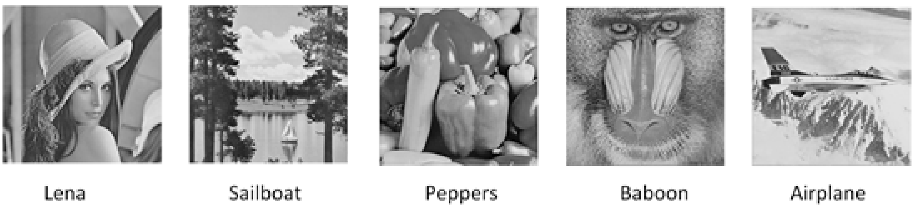

Figure 3: The grayscale original images

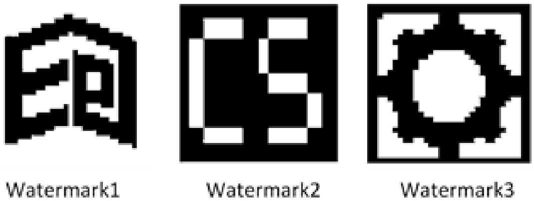

Figure 4: The watermark images

Table 1 shows the PSNR of the watermarked images under no attacks while table 2 shows the NC values of watermark images under no attacks. Generally, a high level of PSNR values indicates that the presented technique provides good performance in invisibility. A high level of NC values indicates that the presented technique provides good performance in robustness against attacks. The presented watermarking technique under no attacks has obtained the NC value of 1.000, and the PSNR values more than 41.923 dB. In visual observations, figures 5-7 observe the watermarked images and extracted watermark images under no attacks.

Table 1: The PSNR values under no attacks

image	Watermark_1	Watermark_2	Watermark_3
Lena	49.520	49.527	49.521
Sailboat	46.456	46.445	46.440
Peppers	48.466	48.470	48.474
Baboon	41.923	41.934	41.942
Airplane	50.976	50.965	50.968

Table 2: The NC values under no attacks

image	Watermark_1	Watermark_2	Watermark_3
Lena	1.000	1.000	1.000
Sailboat	1.000	1.000	1.000
Peppers	1.000	1.000	1.000
Baboon	1.000	1.000	1.000
Airplane	1.000	1.000	1.000

Figure 5: The watermarked images with detected watermark_1 under no attack

Figure 6: The watermarked images with detected watermark_2 under no attack

Figure 7: The watermarked images with detected watermark_3 under no attack

As can be seen from table 2 and figures 5-7, the watermark images are more similar to the original watermark images that refer to the presented technique as more robust against attacks. Furthermore, as can be observed from tables 1-2, and figures 5-7, the host images are more similar to the watermarked images which refer to the presented technique as more invisible. In visual observations, figures 8-10 observe the extracted watermark from Lena image under different attacks whilst tables 3-5 observe the NC values of extracted watermark under various attacks. As can be seen from figures 8-10, the extracted watermark images and the watermark images are more similar with a little distortion in JPEG compression, scaling, and sharpening attacks.

Table 3: The NC values of detected watermark with watermark_1 under different Attacks

Attack	Lena	Sailboat	Peppers	Baboon	Airplane
Flipping	1.000	1.000	1.000	1.000	1.000
Sharpen	0.971	0.961	0.965	0.961	0.958

Attack	Lena	Sailboat	Peppers	Baboon	Airplane
Gamma	1.000	1.000	1.000	1.000	1.000
Blurring	1.000	1.000	1.000	0.999	1.000
Poisson	0.998	0.972	0.970	0.990	0.997
Scaling	0.862	0.840	0.877	0.847	0.856
JPEG	0.818	0.853	0.849	0.892	0.828

From the tables and figures, can be concluded that the presented watermarking technique has a good performance in invisibility and is robust against attacks. To have more evaluation of the presented watermarking technique, the comparison is performed on the presented watermarking technique with the existing technique (31) which utilized similar test images and watermark (watermark_1). The comparison of the presented watermarking technique to the existing technique is demonstrated in table 6 which shows the presented technique has a higher level of invisibility than the existing technique.

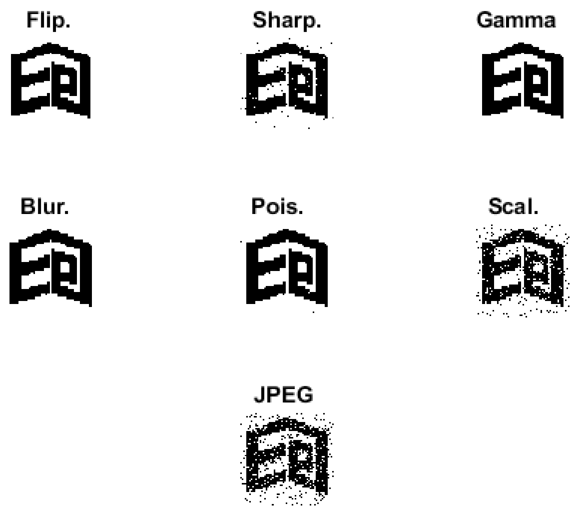

Figure 8: The extracted watermark_1 from Lena image under different attack

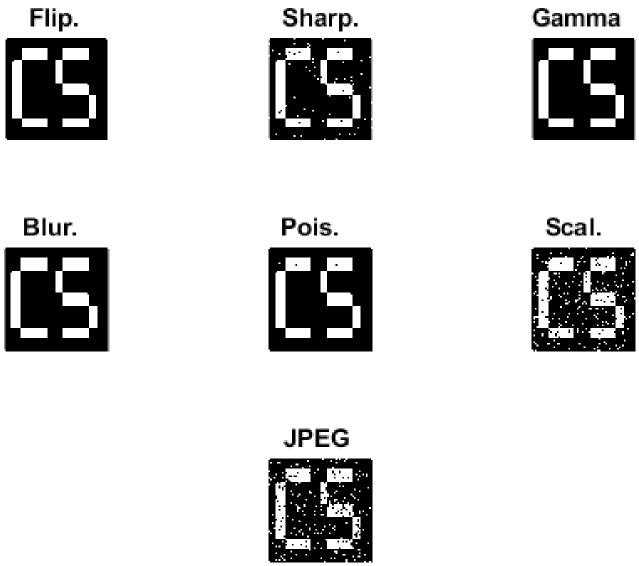

Figure 9: The extracted watermark_2 from Lena image under different attack

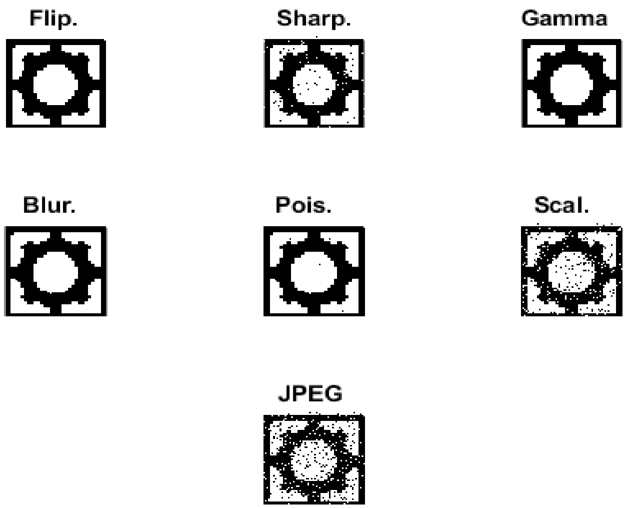

Figure 10: The extracted watermark_3 from Lena image under different attack

Table 4: The NC values of detected watermark with watermark_2 under different Attacks

Attack	Lena	Sailboat	Peppers	Baboon	Airplane
Flipping	1.000	1.000	1.000	1.000	1.000
Sharpen	0.950	0.942	0.943	0.943	0.939
Gamma	1.000	1.000	1.000	1.000	1.000
Blurring	1.000	1.000	1.000	0.999	0.999
Poisson	0.996	0.921	0.966	0.827	0.993
Scaling	0.836	0.816	0.850	0.795	0.822
JPEG	0.815	0.816	0.826	0.839	0.806

Table 5: The NC values of detected watermark with watermark_3 under different Attacks

Attack	Lena	Sailboat	Peppers	Baboon	Airplane
Flipping	1.000	1.000	1.000	1.000	1.000
Sharpen	0.962	0.962	0.966	0.965	0.963
Gamma	1.000	1.000	1.000	1.000	1.000
Blurring	1.000	1.000	1.000	0.999	1.000
Poisson	0.997	0.976	0.977	0.991	0.998
Scaling	0.882	0.854	0.877	0.826	0.864
JPEG	0.838	0.858	0.848	0.884	0.835

Table 6: The comparison of the presented technique of watermark_1 with the existing technique

Image	Scheme	PSNR
Lena	DWT-DCT-SVD (31)	41.970
	Proposed	**49.520**
Sailboat	DWT-DCT-SVD (31)	40.807
	Proposed	**46.456**
Peppers	DWT-DCT-SVD (31)	43.115
	Proposed	**48.466**
Baboon	DWT-DCT-SVD (31)	42.549
	Proposed	**41.923**
Airplane	DWT-DCT-SVD (31)	41.863
	Proposed	**50.976**

5. Conclusion

An optimized image watermarking technique based on LU factorization and entropy analysis in combination with lifting wavelet transform, and discrete cosine transform is presented in this paper for copyright protection of multimedia information. In the proposed technique, various watermarks are hidden in various test images for evaluation of the effects of various watermarks on various test images. Several attacks are performed like flipping, scaling, Gamma correction, JPEG 2000 compression, blurring, adding noise, and sharpening attacks. Four main watermarking properties including imperceptibility, robustness, security, and watermark capacity are obtained in the presented technique, the imperceptibility (PSNR value > 41 dB) under no attacks, the robustness (NC value = 1.000) under no attacks, the security by utilizing random permutation function, and watermark capacity (64^2). The presented watermarking technique is invariant against attacks and has a good performance in imperceptibility, robustness, security, and watermark capacity.

■ References

1. Wang J., Wan W. B., Li X. X., De Sun J.,Zhang H. X., (2020), Color image watermarking based on orientation diversity and color complexity. Expert Systems with Applications, 140, 112868.
2. Mahto D. K., Singh A. K., (2021), A survey of color image watermarking: State-of-

the-art and research directions. Computers & Electrical Engineering, 93, 107255.
3. Mehraj S., Mushtaq S., Parah S. A., Giri K. J., Sheikh J. A., Gandomi A. H., Hijji M.,Muhammad K., (2022), Spatial Domain-Based Robust Watermarking Framework for Cultural Images. IEEE Access, 10, 117248-117260.
4. Basha S. H., (2022), A novel secured Euclidean space points algorithm for blind spatial image watermarking. EURASIP Journal on Image and Video Processing, 2022, 21.
5. AlShaikh M., Alzaqebah M.,Jawarneh S., (2023), Robust watermarking based on modified Pigeon algorithm in DCT domain. Multimedia Tools and Applications, 82, 3033-3053.
6. Byun S.-W., Son H.-S.,Lee S.-P., (2019), Fast and robust watermarking method based on DCT specific location. IEEE Access, 7, 100706-100718.
7. Begum M., Uddin M. S., (2021), Implementation of secured and robust DFT-based image watermark through hybridization with decomposition algorithm. SN Computer Science, 2, 221.
8. Li M., Yuan X., Chen H.,Li J., (2020), Quaternion discrete fourier transform-based color image watermarking method using quaternion QR decomposition. IEEE Access, 8, 72308-72315.
9. Abodena O., Agoyi M.,Celebi E., (2017), *Hybrid technique for robust image watermarking using discrete time fourier transform*, *25th Signal Processing and Communications Applications Conference (SIU)*, 1-4.
10. Salehnia T., Fathi A., (2021), Fault tolerance in LWT-SVD based image watermarking systems using three module redundancy technique. Expert Systems with Applications, 179, 115058.
11. Sharma S., Sharma H.,Sharma J. B., (2021), Artificial bee colony based perceptually tuned blind color image watermarking in hybrid LWT-DCT domain. Multimedia Tools and Applications, 80, 18753-18785.
12. Abodena O., Agoyi M., (2018), Colour image blind watermarking scheme based on fast walsh hadamard transform and hessenberg decomposition. Studies in Informatics and Control, 27, 339-348.
13. Chen S., Su Q., Wang H.,Wang G., (2022), A high-efficiency blind watermarking algorithm for double color image using Walsh Hadamard transform. The Visual Computer, 38, 2189-2205.
14. Zainol Z., Teh J. S.,Alawida M., (2020), A new chaotic image watermarking scheme based on SVD and IWT. Ieee Access, 8, 43391-43406.
15. Zhu T., Qu W.,Cao W., (2022), An optimized image watermarking algorithm based on SVD and IWT. The Journal of Supercomputing, 78, 222-237.
16. Su Q., Chen B., (2017), A novel blind color image watermarking using upper Hessenberg matrix. AEU-International Journal of Electronics and Communications, 78, 64-71.
17. Liu J., Huang J., Luo Y., Cao L., Yang S., Wei D.,Zhou R., (2019), An optimized image watermarking method based on HD and SVD in DWT

domain. IEEE Access, 7, 80849-80860.
18. Su Q., Wang G., Zhang X., Lv G.,Chen B., (2017), An improved color image watermarking algorithm based on QR decomposition. Multimedia Tools and Applications, 76, 707-729.
19. Naderahmadian Y., Hosseini-Khayat S., (2014), Fast and robust watermarking in still images based on QR decomposition. Multimedia tools and applications, 72, 2597-2618.
20. Su Q., Wang G., Zhang X., Lv G.,Chen B., (2018), A new algorithm of blind color image watermarking based on LU decomposition. Multidimensional Systems and Signal Processing, 29, 1055-1074.
21. Jane O., Elbaşı E., (2014), A new approach of nonblind watermarking methods based on DWT and SVD via LU decomposition. Turkish Journal of Electrical Engineering and Computer Sciences, 22, 1354-1366.
22. Hsu L.-Y., Hu H.-T., (2019), A reinforced blind color image watermarking scheme based on Schur decomposition. IEEE Access, 7, 107438-107452.
23. Karajeh H., Khatib T., Rajab L.,Maqableh M., (2019), A robust digital audio watermarking scheme based on DWT and Schur decomposition. Multimedia Tools and Applications, 78, 18395-18418.
24. Singh A. K., (2019), Robust and distortion control dual watermarking in LWT domain using DCT and error correction code for color medical image. Multimedia Tools and Applications, 78, 30523-30533.
25. Abodena O., (2022), A robust blind watermarking scheme based on lifting wavelet transform and hessenberg decomposition. Journal of Pure & Applied Sciences, 21, 48-54.
26. Liu S., Pan Z.,Song H., (2017), Digital image watermarking method based on DCT and fractal encoding. IET image processing, 11, 815-821.
27. Abodena O., (2023), A Robust Blind Grayscale Image Watermarking Technique Based on Schur Decomposition and Entropy Analysis. Gharyan University Journal, 28, 471-493.
28. Weber A. G. (February 2018). *The USC-SIPI image database*. Available: http://sipi.usc.edu/database/
29. Abodena O., (2021), *Robust and high-capacity audio watermarking basedon chirp z-transform*, *29th Signal Processing and Communications Applications Conference (SIU)*, 1-4.
30. Abodena O., Alashtir A., (2021), High Hiding Capacity Audio Watermarking Method Based on Discrete Cosine Transform. Internation Journal Of Advance Research And Innovative Ideas In Education, 7, 677-684.
31. Kang X., Chen Y., Zhao F.,Lin G., (2020), Multi-dimensional particle swarm optimization for robust blind image watermarking using intertwining logistic map and hybrid domain. Soft Computing, 24, 10561-10584.